RCDT RISK ASSESSMENT AND MANAGEMENT

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FAA semi-annual rotorcraft structures R&D and safety review meeting,
February 13-15, 2007

Outline

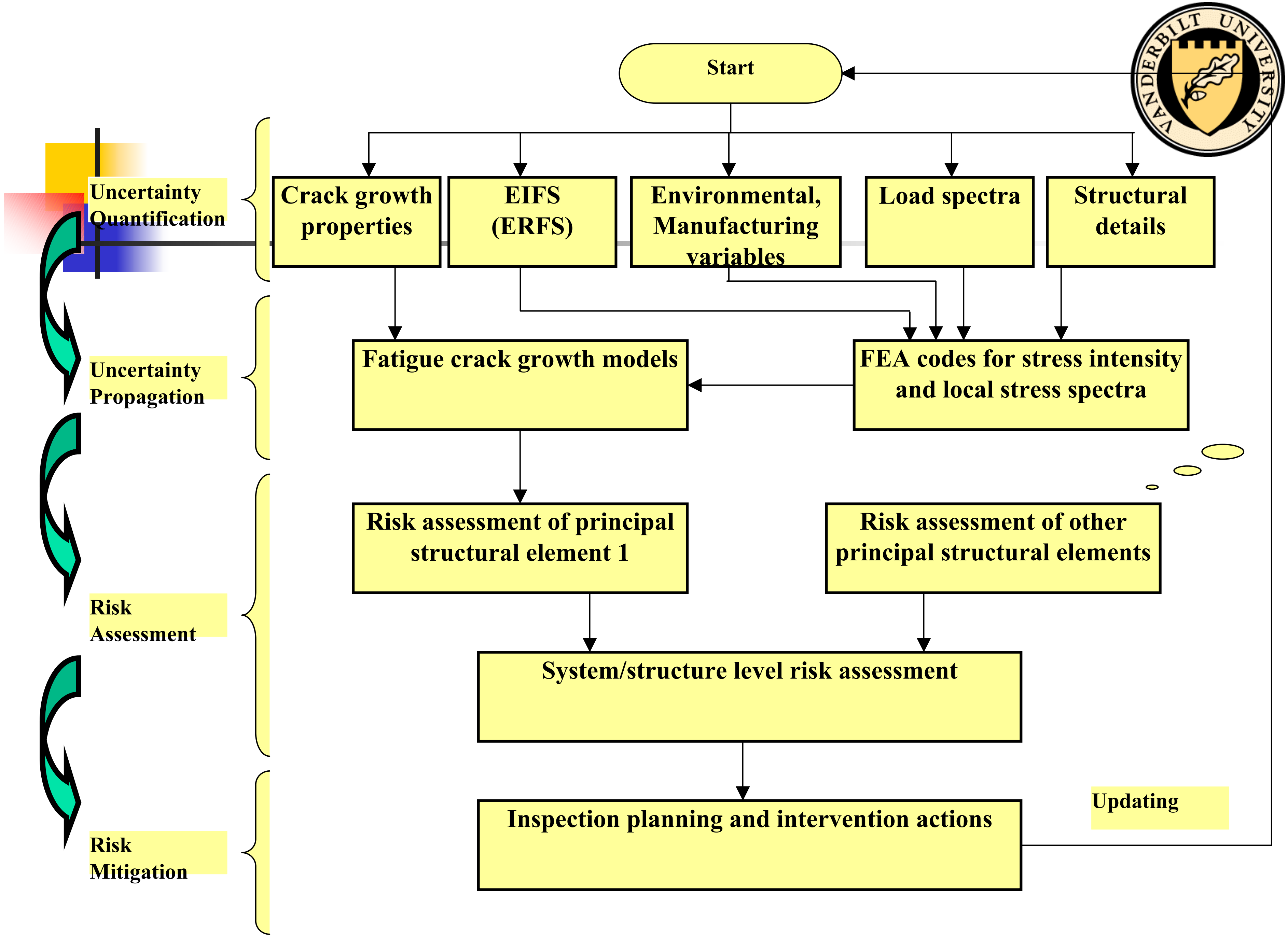
- Project overview
  - Proposed methodology and tasks
  - Work breakdown structure
  - Program schedule
- Accomplishments during the past reporting period (3 months)
  - Uncertainty quantification of FCG properties
  - Probabilistic EIFS calculation
  - Mixed-mode crack propagation
- Conclusions and planned work

Program objectives

- Develop, validate and implement a general risk assessment and management methodology for rotorcraft damage tolerance

- Four major technical objectives
  - Uncertainty quantification at material/specimen level under controlled laboratory conditions
  - Uncertainty propagation to component/structural level under simulated service conditions
  - Risk assessment at system level under actual service conditions
  - Risk management actions

- The results will help FAA rulemaking w.r.t. FAR 29.571, FAR 27.571, AC 29-2A, AC 27-1A, AC-29-2C



Start
Uncertainty Quantification
Crack growth properties
EIFS (ERFS)
Environmental, Manufacturing variables
Load spectra
Structural details
Uncertainty Propagation
Fatigue crack growth models
FEA codes for stress intensity and local stress spectra
Risk Assessment
Risk assessment of principal structural element 1
Risk assessment of other principal structural elements
System/structure level risk assessment
Risk Mitigation
Inspection planning and intervention actions
Updating
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Outline

- Uncertainty Quantification (UQ)
  1. Material properties
  2. Equivalent initial flaw size (EIFS)
  3. Load spectrum
  4. Structural details
  5. Other effects (environmental, manufacturing, etc)
- Uncertainty Propagation (UP)
- Risk Assessment (RA)
- Risk Mitigation (RM)

Material properties – data collection and analysis

- These material properties include but not limit to the following properties

  a) basic mechanical properties (Young's modulus, yielding strength)

  b) classical fatigue properties (S-N, e-N, fatigue limits)

  c) fatigue crack propagation properties (crack threshold value, crack growth curve)

- Different sources of data collection are identified as

  a) Publicly released data, i.e. journal articles, conference proceedings, and technical reports

  b) Previous and current FAA database on RCDT analysis, i.e. FCGD software developed in FAA

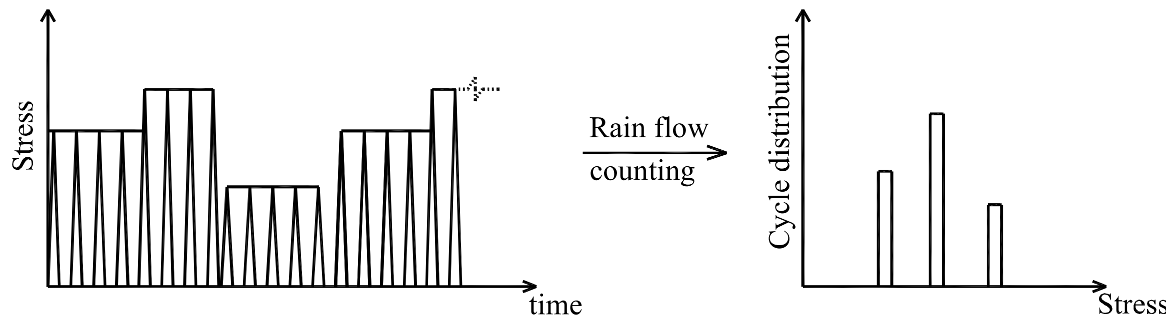
  c) Private experimental data from our subcontractor, Bell Helicopter Inc.

EIFS – method development

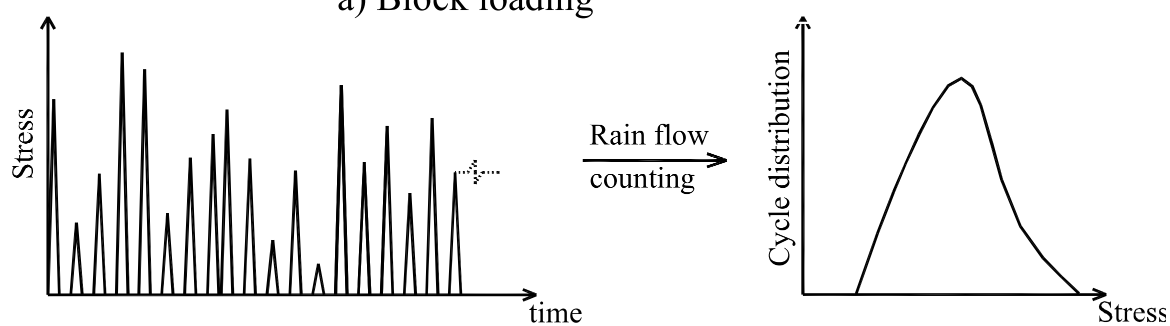
- Data collection for crack growth data and crack initiation data
- Methodology development for efficient calculation of probabilistic EIFS distribution
- Demonstration examples for RCDT reliability evaluation using developed EIFS distribution and fatigue crack growth analysis

Load spectra

- Data collection and analysis
- Frequency domain analysis, e.g. spectral density method
- Time domain analysis, e.g. cycle counting method



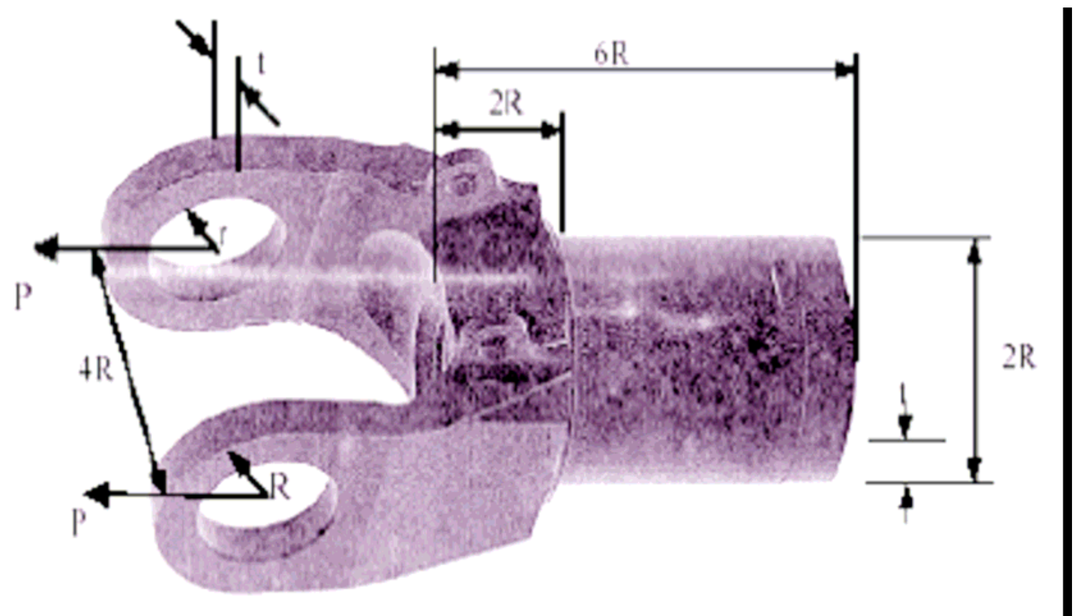
a) Block loading



b) Continuous random loading

Structural details

- Data collection and analysis (help from Bell Helicopter Textron)
- Component geometry (rotor, hug, fuselage, etc)
- Randomness of parameters (thickness, hole radius, etc)
- Random field representation to include spatial randomness, e.g. Karhunen-Loeve expansion



Manufacturing process and environmental effects

- Surface treatment method (residual stress, surface roughness)
- Thermal treatment method (residual stress, material property change)
- Corrosion induced defects
- Foreign-object impact induced defects

Summary for uncertainty quantification

Task 3: Uncertainty quantification (UQ) of basic variables

3.1 UQ of material properties
- 3.1.1 Data collection and analysis
- 3.1.2 Model comparison
- 3.1.3 Bayesian approach for model selection

3.2 UQ of equivalent initial flaw size (EIFS)
- 3.2.1 Crack growth and initiation data collection
- 3.2.2 Model development for EIFS calculation
- 3.2.3 Model demonstration and validation

3.3 UQ of load spectrum
- 3.3.1 Data collection and analysis
- 3.3.2 Comparison of load spectra representing method
- 3.3.3 Demonstration and application in RCDT reliability analysis

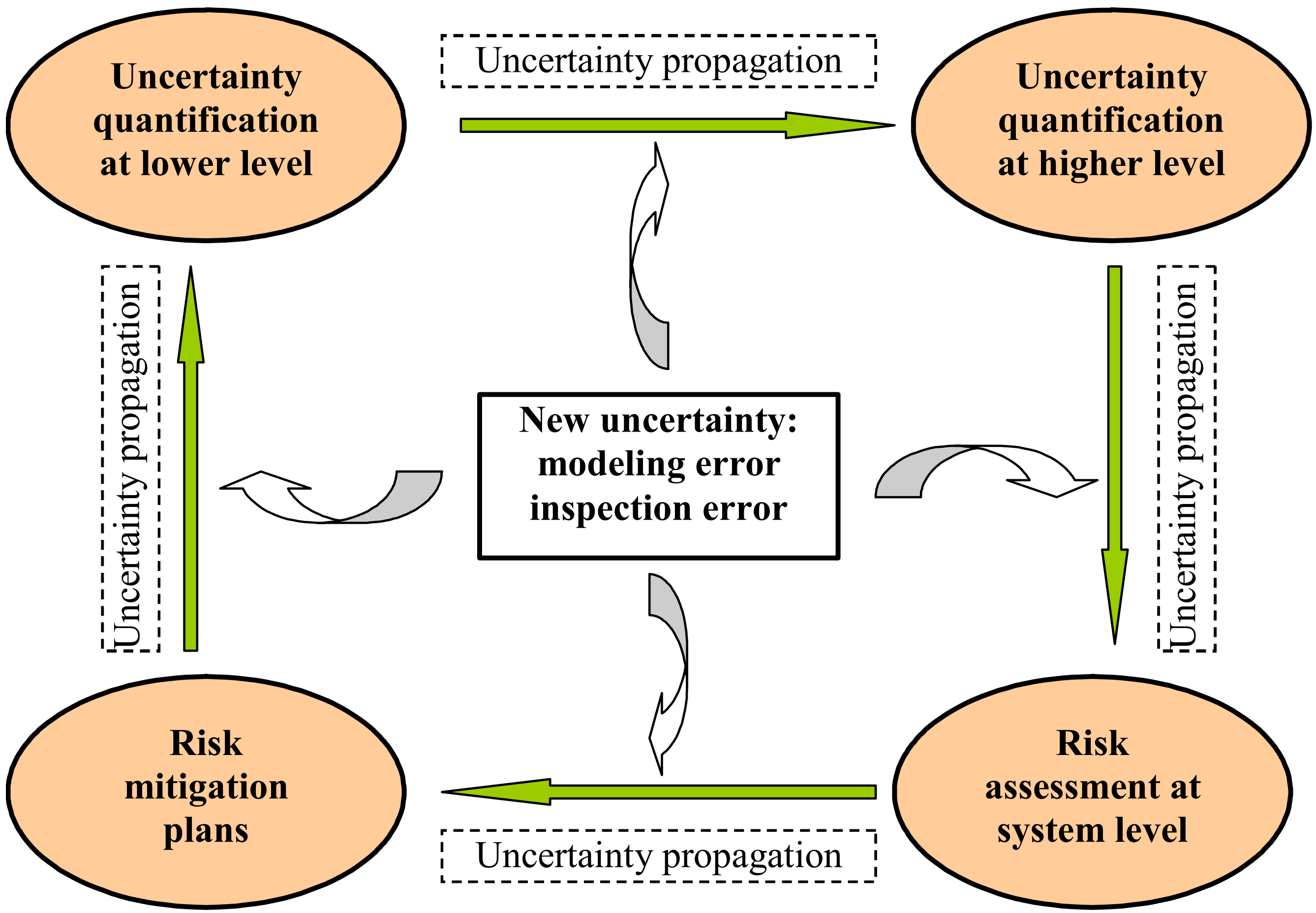
3.4 UQ of structural details and component geometry
- 3.4.1 Structural geometry data collection and analysis
- 3.4.2 Development of computational methodology
- 3.4.3 Demonstration example of the methodology

3.5 UQ of manufacturing process and environmental effects
- 3.5.1 Data collection and analysis
- 3.5.2 Demonstration example using quantified uncertainty
- 3.5.3 Uncertainty updating using future inspection data

Outline

- Uncertainty Quantification (UQ)
- Uncertainty Propagation (UP)
  1. Finite element model (FEM)
  2. Fatigue crack growth (FCG)
  3. Lab to service
  4. Model validation
- Risk Assessment (RA)
- Risk Mitigation (RM)

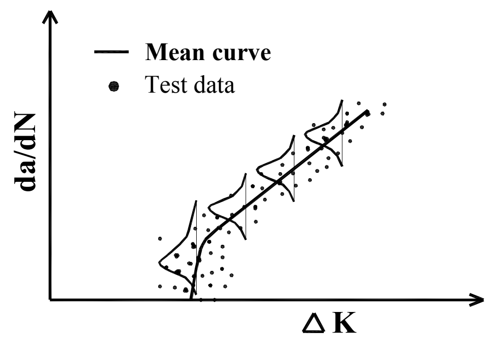
UP - Overview



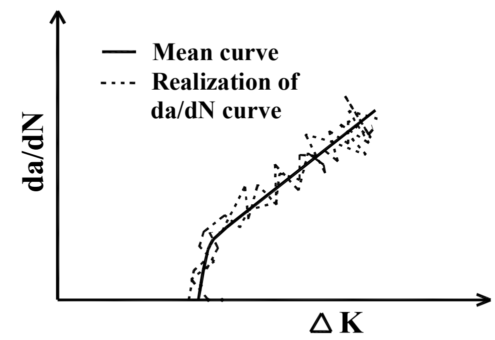
Uncertainty quantification at lower level
Uncertainty propagation
Uncertainty quantification at higher level
Uncertainty propagation
New uncertainty: modeling error inspection error
Uncertainty propagation
Risk mitigation plans
Uncertainty propagation
Risk assessment at system level

Probabilistic crack growth

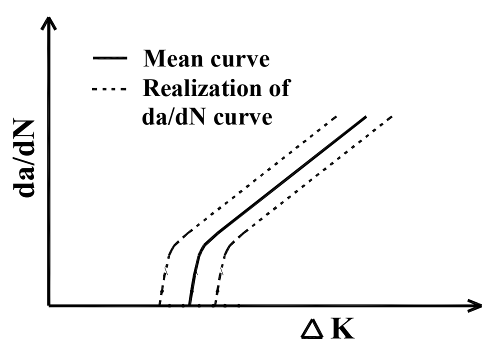
- Crack growth representation



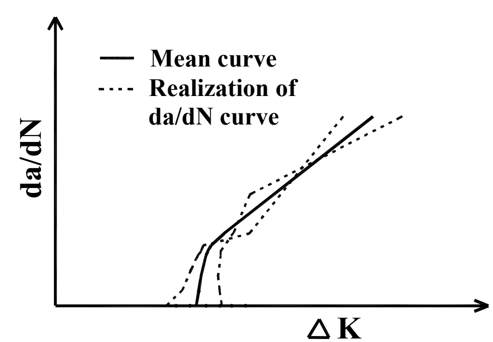
a) da/dN vs ΔK curve of experiments



b) white noise da/dN vs ΔK curve



c) percentile da/dN vs ΔK curve



b) stochastic da/dN vs ΔK curve

UP – Covariance structure

- Importance of covariance structure
  - Example of crack growth during two stress cycles

$$\Delta a_{total} = \Delta a_1 + \Delta a_2 = (\frac{da}{dN})_{\Delta K_1} \times 1 + (\frac{da}{dN})_{\Delta K_2} \times 1$$

$$mean(\Delta a_{total}) = mean(\Delta a_1) + mean(\Delta a_2)$$

$$Var(\Delta a_{total}) = Var(\Delta a_1) + Var(\Delta a_2) + 2\rho\sqrt{Var(\Delta a_1)Var(\Delta a_2)}$$

Covariance structure of crack growth curve will not affect deterministic prediction (mean value) but affect the reliability prediction (variance).

UP – Random process representation of crack growth

- Probabilistic crack growth prediction needs to consider the covariance/correlation among the basic input random variables
- Crack growth process is treated as a random process using Karhunen-Loeve random process expansion technique

$$log(\frac{da}{dN}) = \sigma_{log(N)}(\Delta K)\sum_{i=1}^{\infty}\sqrt{\lambda_i}\xi_i(\theta)f_i(\Delta K) + log(\overline{\frac{da}{dN}(\Delta K)})$$

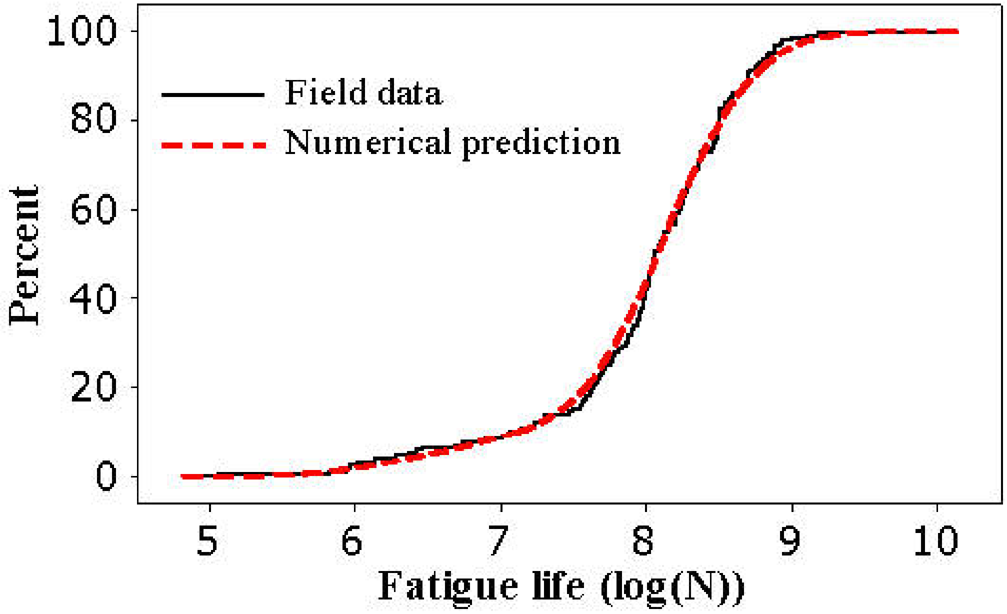
$\xi_i(\theta)$ set of independent standard Gaussian random variables

$\sqrt{\lambda_i}$ $f_i(x)$ *ith* eigenvalues and eigenfunctions of the covariance function

$C(\Delta K_1, \Delta K_2) = e^{-\mu|\Delta K_1 - \Delta K_2|}$ covariance function

UP – model validation

- Probabilistic life prediction
- Statistical model validation (i.e., classical hypothesis testing, Bayesian factor)



Summary for uncertainty propagation

Task 4: Uncertainty propagation (UP) through numerical models

- 4.1 UP through finite element models
  - 4.1.1 Develop and validate appropriate numerical model
  - 4.1.2 Propagate quantified uncertainty through the developed model
  - 4.1.3 Demonstration example
- 4.2 UP through fatigue crack growth models
  - 4.2.1 Develop and validate a general FCG model
  - 4.2.2 Propagate quantified uncertainties through the FCG model
  - 4.2.3 Demonstration example for probabilistic crack growth prediction
- 4.3 UP from laboratory conditions to service conditions
  - 4.3.1 Data collection and analysis under service conditions
  - 4.3.2 Simulate the service condition using developed numerical model
  - 4.3.3 Sensitivity analysis of critical factors
  - 4.3.4 Model calibration and updating using inspection data
- 4.4 Model validation
  - 4.4.1 Qualitative comparison and validation of model prediction
  - 4.4.2 Quantitative model validation using Bayesian statistics

Outline

- Uncertainty Quantification (UQ)
- Uncertainty Propagation (UP)
- Risk Assessment (RA)
  1. Analytical and numerical
  2. DOE and RSM
  3. Sensitivity analysis
  4. Life prediction and validation
- Risk Mitigation (RM)

Analytical and numerical methods

- Limit state function *g(x),* e.g. accumulated crack growth is less than a critical value, time to a certain crack length is less than a predefined life

$$P_f = \int \cdots \int_{g(\mathbf{x})<0} f_{\mathbf{x}}(\mathbf{x}) d\mathbf{x}$$

- Analytical – First Order Reliability Method (FORM), Second Order Reliability Method (SORM)

FORM $P_f = \Phi(-\beta)$

β first-order reliability index

SORM $P_f = \Phi(-\beta)\prod_{i=1}^{n-1}\left(1+\beta\kappa_i\right)^{-1/2}$

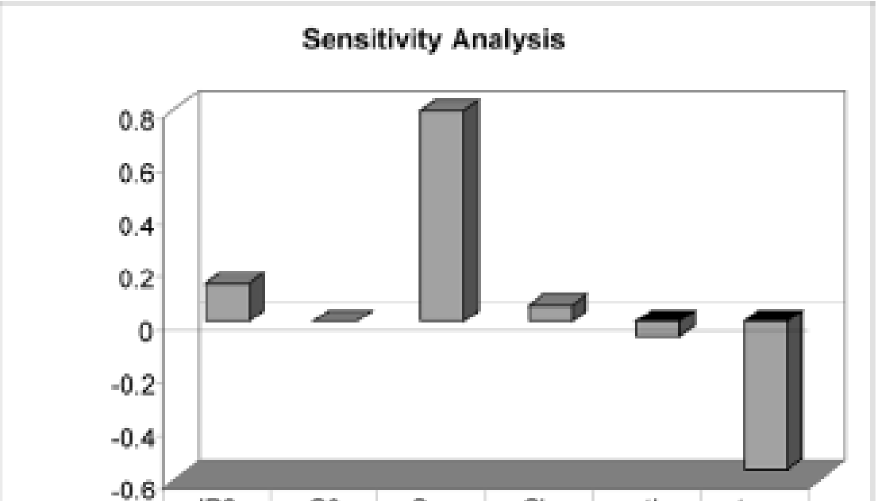
κ_i main curvatures of the failure surface at the design point

DOE and RSM

- Direct reliability analysis using FEM is computationally prohibitive
- Response Surface Method (RSM) is to construct an approximate closed-form mathematical relationship between input variables and output variables using a few sample points
- Input sample points are chosen based on the Design of Experiments (DOE)
- Output response are calculated using developed numerical models and fitted using least-square regression

Sensitivity analysis

- Recommendations for future design, e.g. reduce uncertainties of critical variables
- Safety factors for different design variables
- Sensitivity factors: gradient of the limit state function at the most probable point (MPP)
- Analytical differentiation or finite difference method



	IP0	C0	Csc	Clc	ath	tpn
sensitivity	0.141	5.4E-05	0.798	6.1E-02	-5.6E-02	-0.561

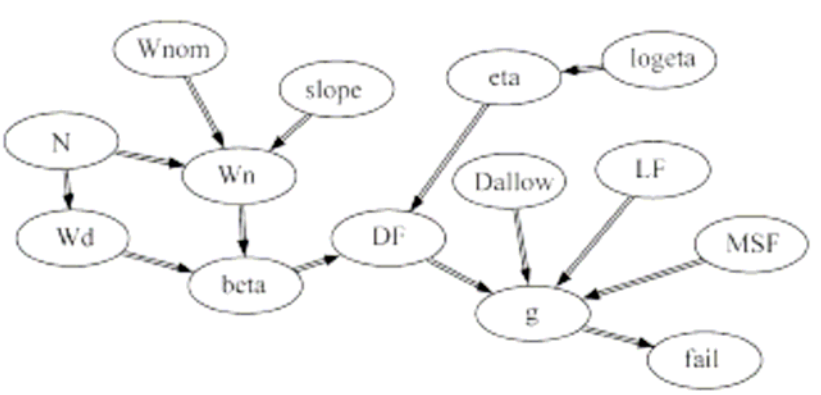
Model validation

- Model validation involves comparison of two or more uncertain quantities
- Need to answer whether the model is close enough to the data (accuracy requirement)
- Assess whether the degree of confidence in the accuracy is sufficiently high (adequacy requirement)

Various metrics for statistical validation -- based on *p*-values, confidence intervals, probability intervals, Bayes factors, etc.

Model validation – system level

- System level validation is difficult
- No or very few full-scale data
- Bayesian network to derive system validation based on sub-module results



Bayes network for an engine blade under high cycle fatigue

Summary for risk assessment

Task 5: Risk Assessment

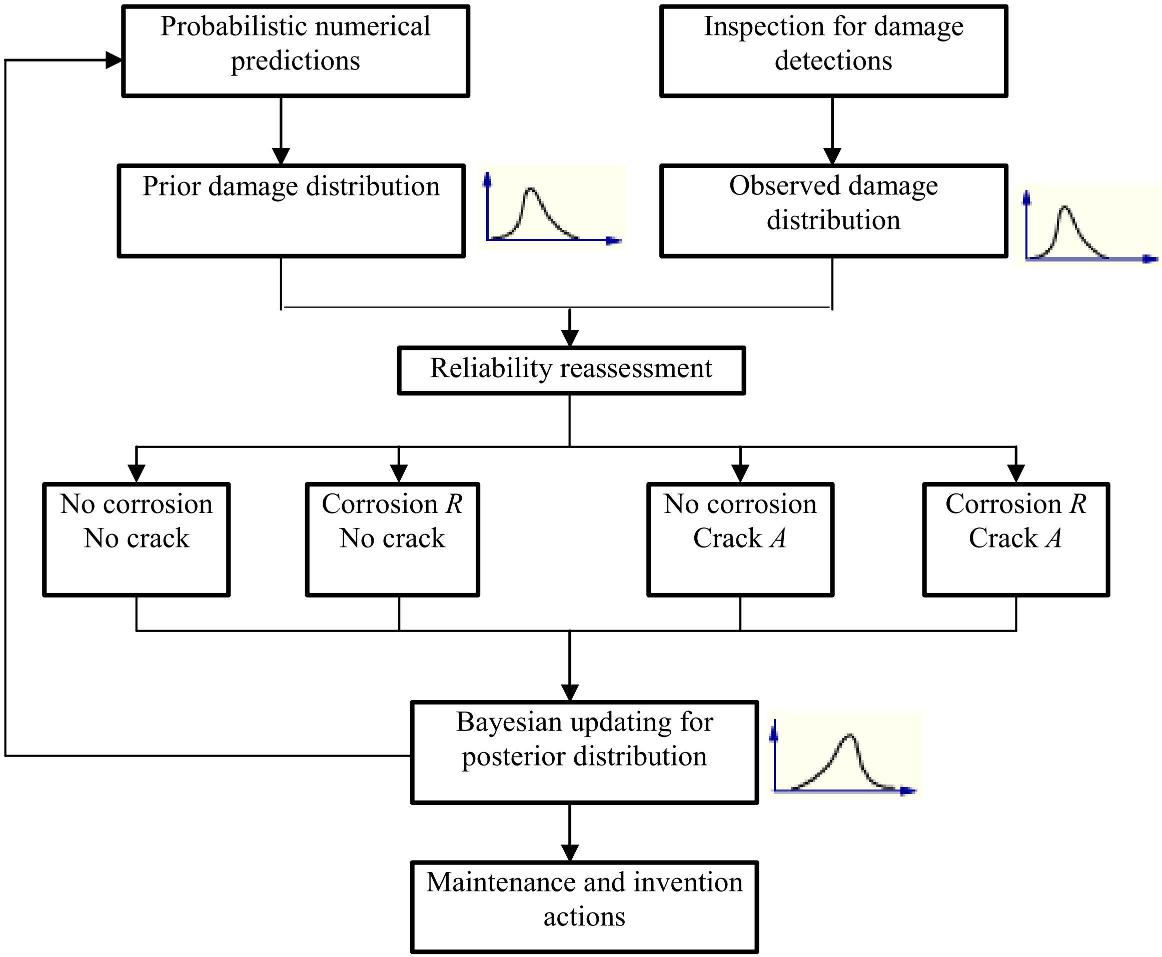
- 5.1 Analytical and numerical methods
  - 5.1.1 Model development for single site risk assessment
  - 5.1.2 Model development for multiple site risk assessment
  - 5.1.3 Demonstration example for RCDT analysis
- 5.2 Design of experiments and response surface
  - 5.2.1 DOE for RCDT analysis
  - 5.2.2 Numerical simulation and response surface approximation
  - 5.2.3 Model calibration and validation
- 5.3 Sensitivity analysis of random variables
  - 5.3.3 Analytical and numerical sensitivity analysis
  - 5.3.4 Parametric study and critical factors identification
  - 5.3.5 Model simplification using sensitivity analysis results
- 5.4 Model validation
  - 5.4.1 System level model validation and verification
  - 5.4.2 Demonstration example

Outline

- Uncertainty Quantification (UQ)
- Uncertainty Propagation (UP)
- Risk Assessment (RA)
- Risk Mitigation (RM)
  1. Reliability assessment using inspection data
  2. inspection interval optimization
  3. inspection location optimization
  4. damage classification and intervention action

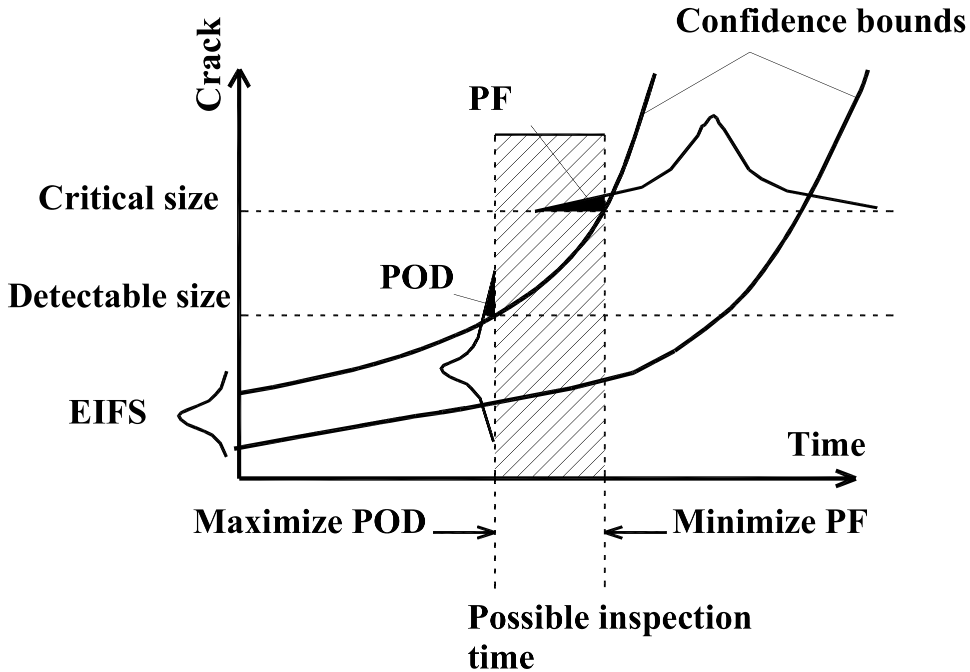
Reliability assessment using inspection data

- Prior distribution from numerical models
- Inspection observations to calibrate, validate and update numerical model
- Integrated approach for continuous reliability evaluation



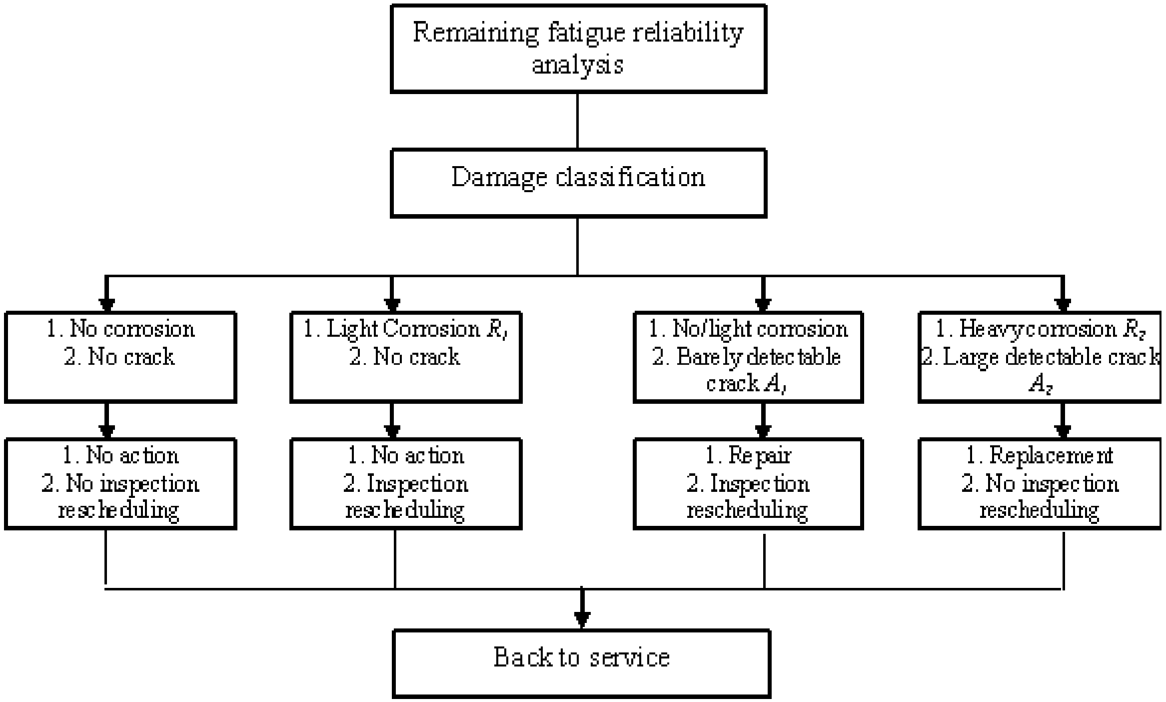
Inspection interval optimization

- Probability of detection (POD) of each inspection
- Optimize the interval to minimize the failure probability and maximize the probability of detection



Damage classification

- Damage classification based on the risk level
- Different intervention actions recommended for different damage stages
- Satisfy both economical and safety constraints



Concept of ERFS

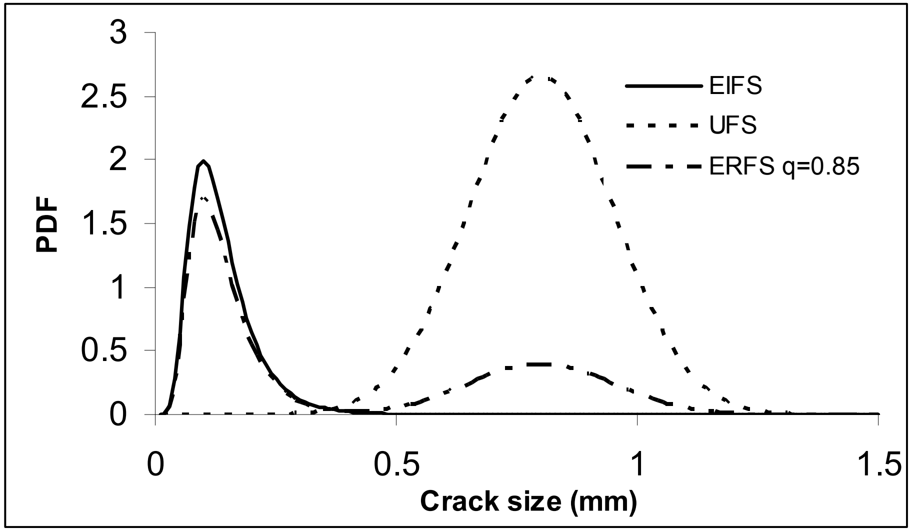
ERFS - Equivalent repair flaw size
EIFS - Equivalent initial flaw size
UFS - Updated flaw size after inspection

-- Bimodal distribution

-- Considered repair quality, *q*

-- EIFS, ERFS, UFS can be used in the proposed methodology for repeated life-cycle analysis

$$P(ERFS) = qP(EIFS) + (1-q)P(UFS)$$



Summary for risk management

Task 6: Risk management

- 6.1 Fatigue reliability updating with inspection data
  - 6.1.1 Methodology development for reliability updating
  - 6.1.2 Data collection and analysis
  - 6.1.3 Demonstration example
- 6.2 Inspection schedule optimization
  - 6.2.1 Reliability-based inspection planning
  - 6.2.2 Data collection and analysis for POD
  - 6.2.3 Inspection schedule optimization under uncertainty
- 6.3 Inspection location optimization
  - 6.3.1 Critical location/components identification from numerical models
  - 6.3.2 Data collection and analysis
  - 6.3.3 Inspection location optimization using sampling techniques
- 6.4 Damage state classification and intervention action recommendation
  - 6.4.1 Methodology development for ERFS
  - 6.4.2 Damage classification and intervention action
  - 6.4.3 Demonstration example

Overall program schedule

Tasks	Year 1				Year 2				Year 3				Year 4				Year 5			
	Q1	Q2	Q3	Q4	Q1	Q2	Q3	Q4	Q1	Q2	Q3	Q4	Q1	Q2	Q3	Q4	Q1	Q2	Q3	Q4
1. Detailed work plan	■																			
2. Reports	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■
3. Uncertainty Quantification	■	■	■	■	■	■	■	■	■	■			■	■	■	■	■	■	■	■
3.1	■	■	■	■																
3.2	■	■	■	■																
3.3			■	■	■	■	■						■	■	■	■				
3.4					■	■	■	■	■											
3.5						■	■	■	■	■					■	■	■	■	■	■
4. Uncertainty Propagation	■	■	■	■	■	■	■	■	■	■	■	■	■	■			■	■	■	■
4.1			■	■	■	■	■	■	■	■										
4.2	■	■	■	■	■	■	■	■	■	■	■	■								
4.3								■	■	■	■	■	■							
4.4			■	■	■	■					■	■	■	■			■	■	■	■
5. Reliability Assessment			■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■		
5.1			■	■	■	■	■	■	■	■	■	■	■	■	■	■				
5.2							■	■	■	■	■	■	■	■						
5.3									■	■	■	■	■	■	■	■				
5.4					■	■	■	■	■	■			■	■	■	■	■	■		
6. Reliability Management									■	■	■	■	■	■	■	■	■	■	■	■
6.1									■	■	■	■	■	■			■	■	■	■
62													■	■	■	■	■	■		
6.3														■	■	■	■	■	■	■
6.4											■	■	■	■	■	■	■	■	■	■
7. Technology transfer plan			■	■			■	■			■	■			■	■			■	■
8. Data Package																			■	■

Current activities and achievements

- Progress from November 1, 2006 – February 1, 2007

  Task 3.1 Uncertainty quantification of FCG properties

  Task 3.2 Probabilistic EIFS calculation

  Task 4.2 Mixed-mode crack propagation

Uncertainty quantification of FCG properties

- Model selection in representing fatigue crack growth curve data
- Randomize the deterministic FCG curve by setting model parameters as random variables
- Point-to-point approach to calculate empirical CDF of random variables
- Anderson-Darling statistical metric to select the best probability distribution

FCG equation

- Walker's equation

$$\frac{da}{dN} = C\left(\frac{\Delta K}{(1-R)^{1-p}}\right)^{m}$$

- NASGRO

$$da/dN = C\left[\left(\frac{1-f}{1-R}\right)\Delta K\right]^{n}\frac{\left(1-\frac{\Delta K_{th}}{\Delta K}\right)^{p}}{\left(1-\frac{K_{max}}{K_{crit}}\right)^{q}}$$

$$f = \frac{K_{op}}{K_{\max}} = \begin{cases} \max\left(R, A_o + A_1R + A_2R^2 + A_3R^3\right) & R \geq 0 \\ A_o + A_1R & -2 \leq R \leq 0 \\ A_o - 2A_1 & R \leq -2 \end{cases}$$

$$A_0 = \left(0.825 - 0.34\alpha + 0.05\alpha^2\right)\left[\cos\left(\frac{\Pi}{2}S_{\max}/\sigma_0\right)\right]^{\frac{1}{\alpha}}$$

$$A_2 = 1 - A_0 - A_1 - A_3 \qquad \Delta K_{th} = \Delta K_0\left(\frac{a}{a+a_o}\right)^{\frac{1}{2}} \Big/ \left(\frac{1-f}{(1-A_0)(1-R)}\right)^{1-C_{th}R}$$

- Simple and easy to quantify uncertainties
- Not suitable for near threshold crack growth

- Comprehensive model and works for all regions of FCG
- Many parameters and hard to quantify uncertainties simultaneously

Anderson-Darling (A-D) Statistic

$$A_n^2 = n\int_{-\infty}^{+\infty} [F_n(x) - F(x)]^2 \varphi(x) f(x)dx$$

n = total number of data points

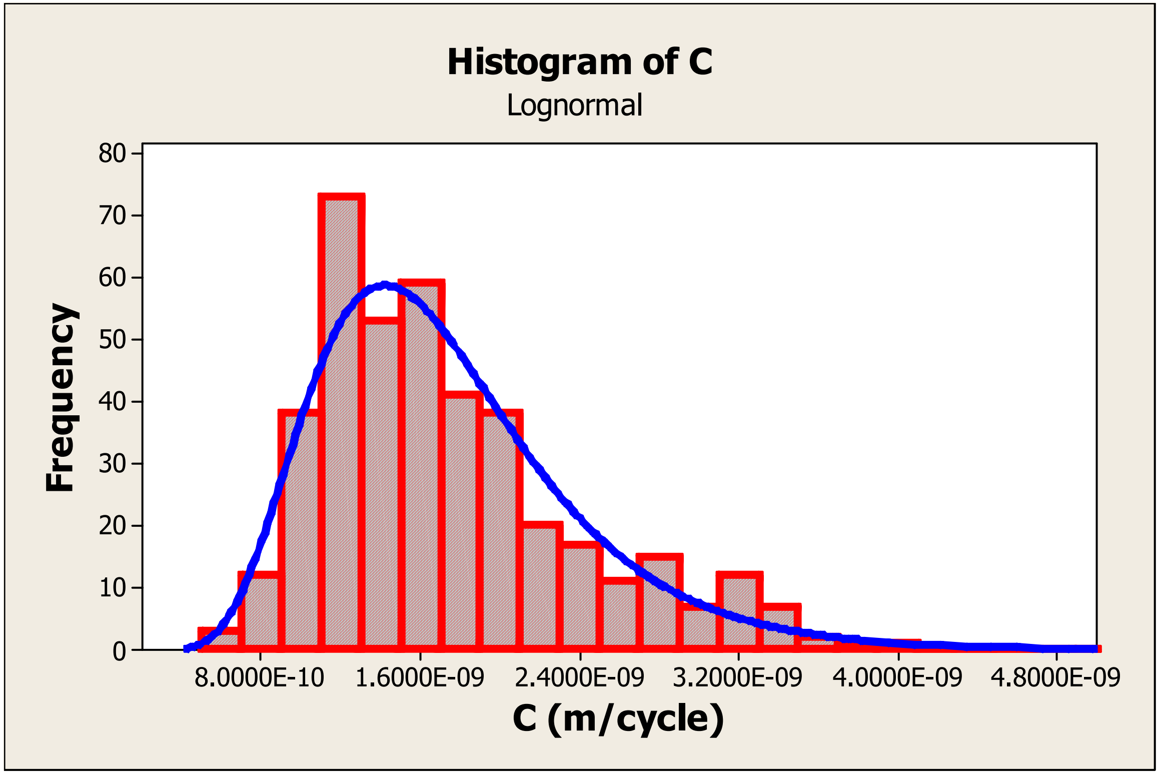
$f(x)$ = *the hypothesized density function*

$F(x)$ = *the hypothesized cumulative distribution function*

$$F_n(x) = \frac{N_x}{n} \qquad \varphi^2(x) = \frac{1}{F(x)[1 - F(x)]}$$

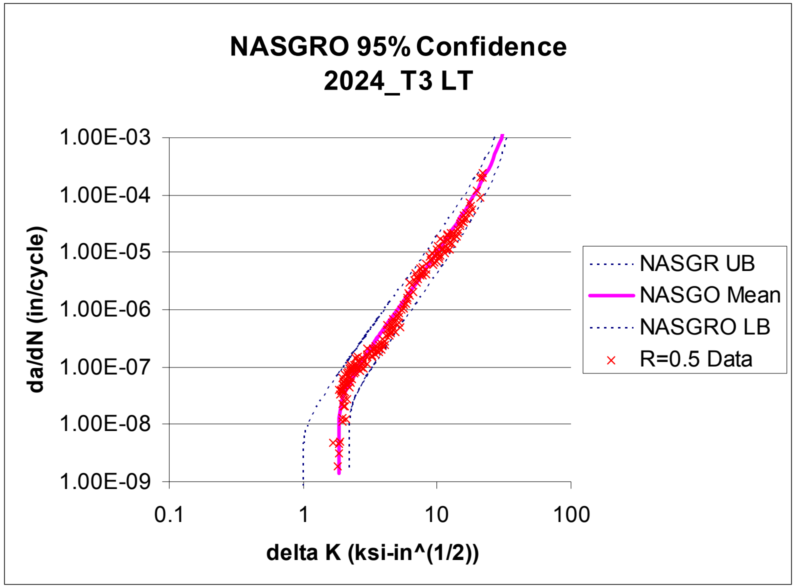
- A-D statistic highlights differences between the tails of the fitted distribution and input data, which is important for structural reliability analysis

Histogram and probability distribution fit

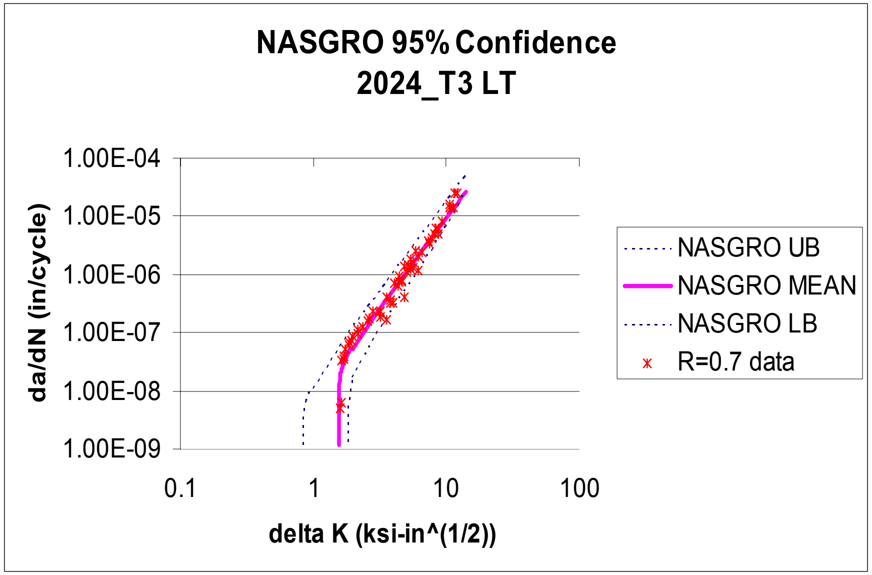


Al 2024 T3

Comparison with experimental data



Al 2024 T3
R=0.5



Al 2024 T3
R=0.7

Summary of data analysis results

ID	Material Name	Type	Orient	Stress Ratios	m	p
1	2014-T6	Sht	T-L	0.05, 0.25, 0.4	3.575	0.596
2	2024-T3	CB Sht	L-T	0.0, 0.5, 0.7	3.273	0.618
3	2024-T3	CB Sht	L-T	-2, -1, -0.5	3.301	0.164
4	2024-T3	CB Sht	T-L	0.0, 0.5, 0.33, 0.4, 0.7, 0.8	3.477	0.623
5	7050_T7451	Plt	L-T	0.08, 0.1, 0.4, 0.5, 0.7, 0.8	3.613	0.538
6	7075_T651	Plt	L-T	0.02, 0.1, 0.33, 0.5, 0.75	2.791	0.65
7	7075-T7351	Plt	L-T	0.1, 0.33, 0.5, 0.8	3.157	0.624
8	7075-T7351	Plt	L-T	-1	3.157	0.071

	--------------WALKER C-------------				
ID	Distibution (parameter1, parameter2)	A-D	Mean	Std	95% Bounds
1	Lognormal (-20.02, 0.1464)	0.4062	2.05E-09	3.19E-10	[1.61e-9 , 2.83e-9]
2	Lognormal (-20.24, 0.3685)	0.6143	1.74E-09	6.85E-10	[8.18e-10 , 3.44e-9]
3	Weibull (2.845, 2.84e-9)	0.8719	1.82E-09	7.02E-10	[8.79e-10 , 3.13e-9]
4	Weibull (1.9404, 8.62e-10)	0.5307	1.27E-09	4.11E-10	[6.37e-10 , 2.19e-9]
5	Weibull (1.4412, 5.11e-10)	0.2678	8.26E-10	3.27E-10	[4.02e-10 , 1.62e-9]
6	Lognormal (-18.243, 0.2402)	0.9143	1.47E-08	3.00E-09	[9.87e-9 , 2.155e-8]
7	Lognormal (-16.965 , 0.1148)	2.936	1.62E-08	4.97E-09	[7.24e-9 , 2.668e-8]
8	Weibull (1.3795, 1.0041e-8)	0.7476	2.04E-08	6.73E-09	[1.197e-8 , 3.713e-8]

Current activities and achievements

- Progress from November 1, 2006 – February 1, 2007

  Task 3.1 Uncertainty quantification of FCG properties

  Task 3.2 Probabilistic EIFS calculation

  Task 4.2 Mixed-mode crack propagation

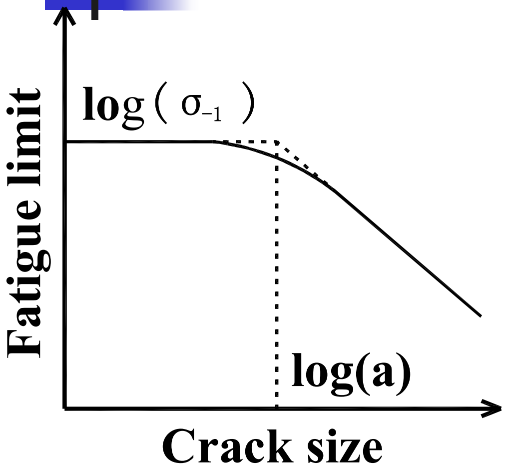
EIFS calculation

- Inspection equipment limits (too conservative)
- Backward extrapolation using fracture mechanics-based FCG analysis (depending on stress level and computationally expensive for probabilistic analysis)
- Two characteristics of a sound EIFS calculation methodology

  - Material property; should be independent of applied mechanical load level

  - Micro-structural property; should be independent of temperature (as long as the temperature is below the transit temperature)

Model development - Kitagawa diagram and El Haddad model

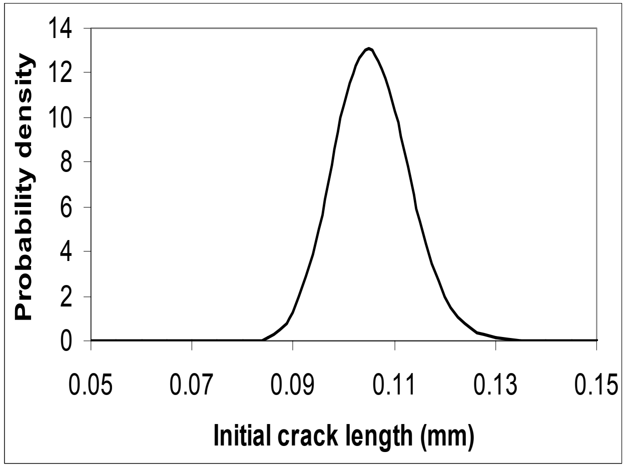


Kitagawa diagram

Random variable

$$a = \frac{1}{\pi}\left(\frac{K_{th}}{f_{-1}}\right)^2$$

Random variable



EIFS distribution

EL Haddad model

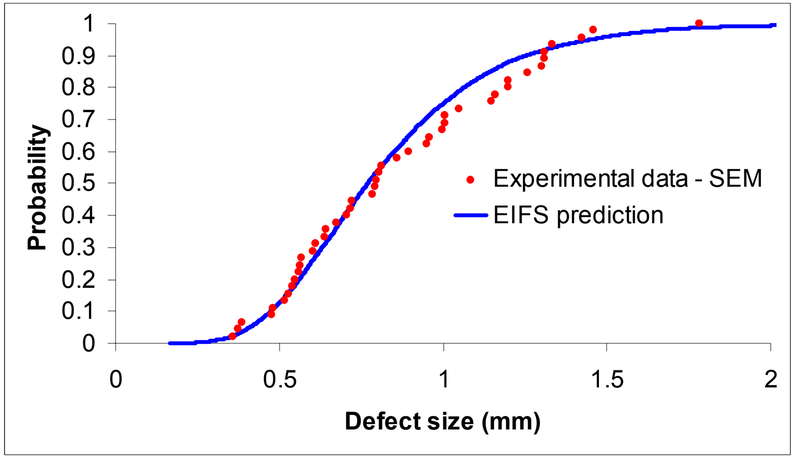
$$f_{-1} = \frac{K_{I,th}}{\sqrt{\pi a}}$$

Experimental data

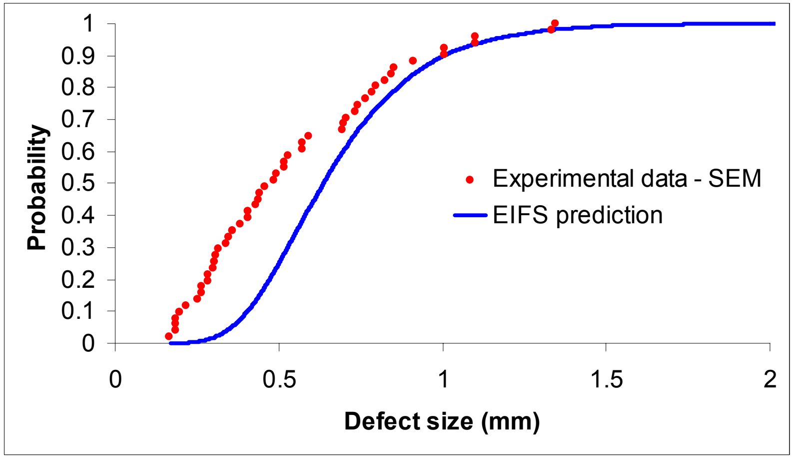
Materials	Fatigue limit Mean ± standard deviation (MPa)	Fatigue crack threshold stress intensity factor Mean ± standard deviation ($MPam^{1/2}$)
AlSi9Cu3	75± 14 (20˚C) 61± 12 (150˚C)	2.57± 0.12 (20˚C) 2.07± 0.12 (150˚C)
AZ91 hp	45± 7 (20˚C) 41± 6 (125˚C)	1.41± 0.12 (20˚C) 1.12± 0.07 (125˚C)
AS21 hp	38± 8 (20˚C) 27± 5 (125˚C)	1.36± 0.11 (20˚C) 1.05± 0.09 (125˚C)

Material	EIFS prediction Mean ± standard deviation (mm)		Experimental data by SEM Mean ± standard deviation (mm)
	Fully dependent assumption	Independent assumption	
AlSi9Cu3	0.80 ± 0.23 (20˚C) 0.79 ± 0.22 (150˚C)	0.83 ± 0.33 (20˚C) 0.82 ± 0.35 (150˚C)	0.89 ± 0.34
AZ91 hp	0.64 ± 0.09 (20˚C) 0.50 ± 0.09 (125˚C)	0.69 ± 0.25 (20˚C) 0.51 ± 0.16 (125˚C)	0.61 ± 0.30
AS21 hp	0.87 ± 0.23 (20˚C) 1.00 ± 0.20 (125˚C)	0.93 ± 0.44 (20˚C) 1.07 ± 0.45 (125˚C)	0.74 ± 0.30

EIFS and realistic defect size



AlSi9Cu3



AZ91 hp

Probabilistic crack growth

- LEFM-based crack growth

$$da/dN = C\Delta K^n \left(1 - \frac{\Delta K_{th}}{\Delta K}\right)^p \qquad N = \int_0^N dN = \int_{a_i}^{a_c} \frac{1}{C\Delta K^n \left(1 - \frac{\Delta K_{th}}{\Delta K}\right)^p} da$$

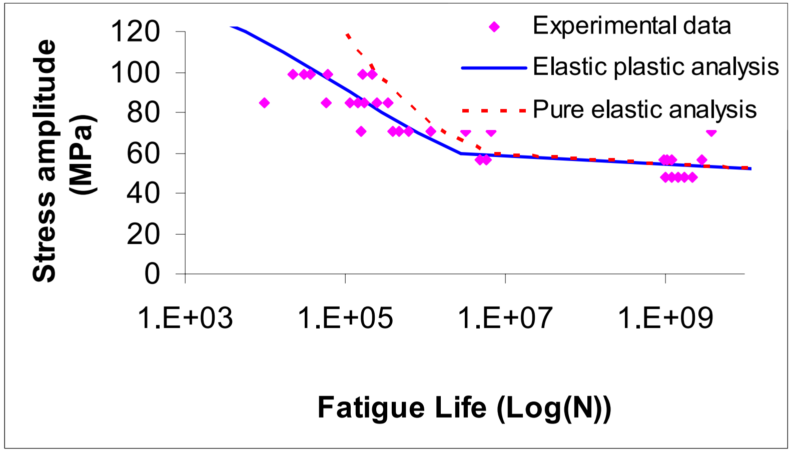
$$K = \sigma\sqrt{\pi a} F\left(\frac{a}{W}\right)$$

- Plasticity correction

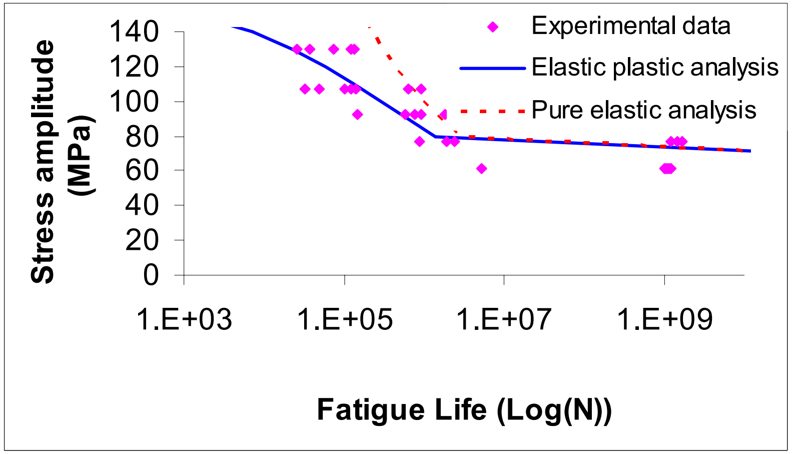
$$K = \sigma \sqrt{\pi a'} F\left(\frac{a'}{W}\right) \qquad a' = a + \rho$$

$$\rho = a\left(sec \frac{\pi\sigma_{max}(1-R)}{4\sigma_0} - 1\right) \qquad \sigma_0 = \left(\frac{\sigma_y + \sigma_u}{2}\right)$$

Elastic vs. plastic prediction



AlSi9Cu3 -20c



AlSi9Cu3 -150c

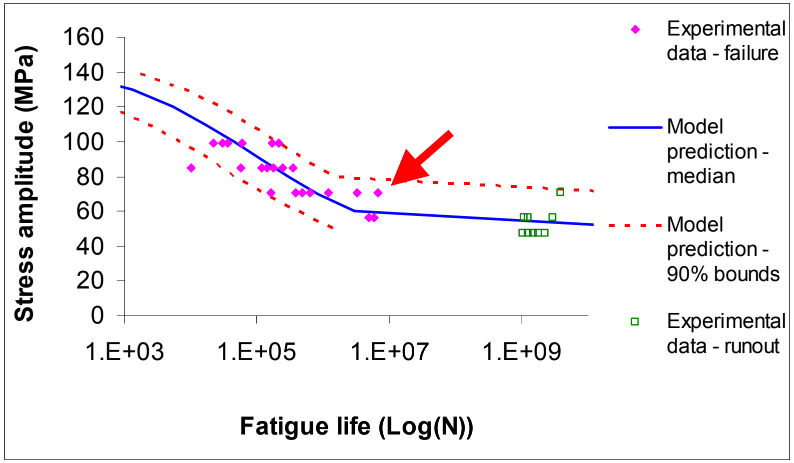
- Plasticity correction is necessary as pure elastic analysis gives a non-conservative prediction
- Differences between elasto-plastic and pure elastic analysis increase as the applied load increases

Calculation parameters and methods

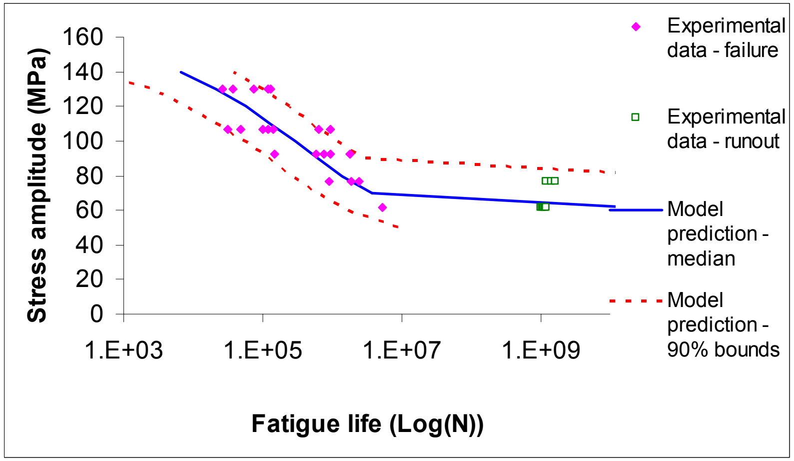
	AlSi9Cu3 at 150˚C			AlSi9Cu3 at 20˚C		
	Mean	Std	Distribution	Mean	Std	Distribution
Yielding strength (MPa)	123	0		134	0	
Ultimate strength (MPa)	198	0		216	0	
C	6.64E-12	3.78E-12	Lognormal	1.79E-12	1.10E-12	Lognormal
n	4.469	0		4.2511	0	
ΔK_{th}	2.07	0.12	Lognormal	2.57	0.12	Lognormal
p	0.75	0		0.25	0	

- Experimental SN curve data are obtained using dog-bone specimen
- All cracks are assumed to be surface crack with half length of a_i and the final critical crack length a_c is assumed to be half of the specimen width
- 10000 Monte Carlo simulation is used to calculate the probabilistic life distribution.

Life prediction comparison



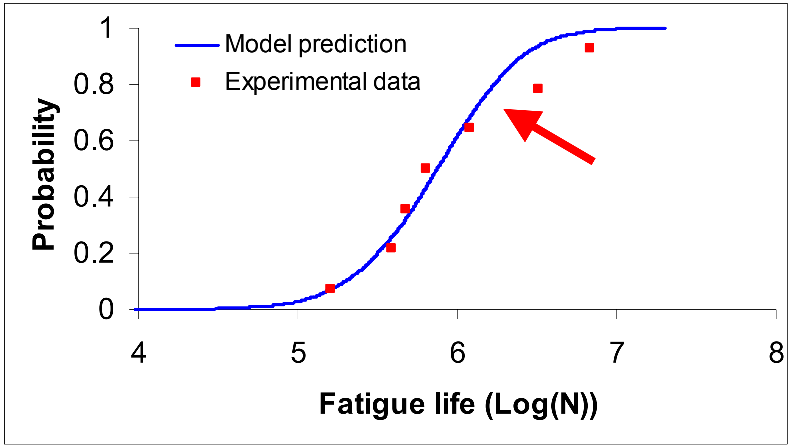
AlSi9Cu3 -20c

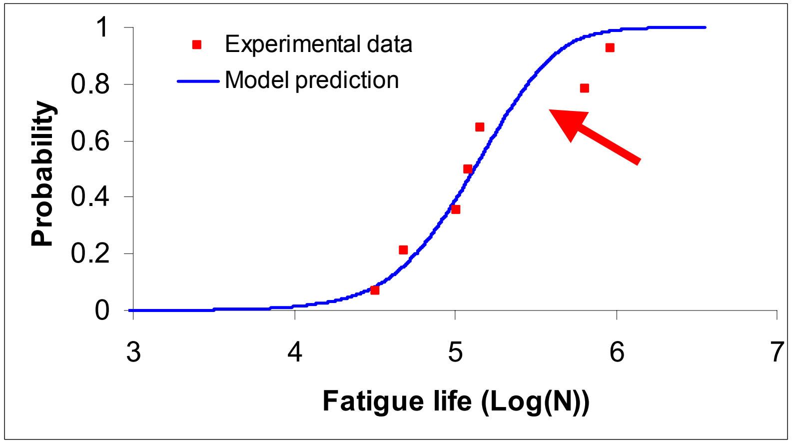


AlSi9Cu3 -150c

- Experimental data: 1 out of 8 specimen experiences run out; failure probability is 87.5%.
- Monte Carlo simulation: 1300 out of 10000 simulations experience run out; failure probability is 87%.

CDF comparison for finite fatigue life distribution





AlSi9Cu3 -20c, S=71MPa

AlSi9Cu3 -150c, S=107MPa

- Capture the major trend of fatigue life distribution
- Some differences at the long life tail region
- Bi-modal distribution and indicate that another failure mechanism is ignored in the model

Internal crack growth

- Stress intensity factor correction for internal crack

$$K = \frac{1}{1.1} \sigma \sqrt{\pi a'} F\left(\frac{a'}{W}\right)$$

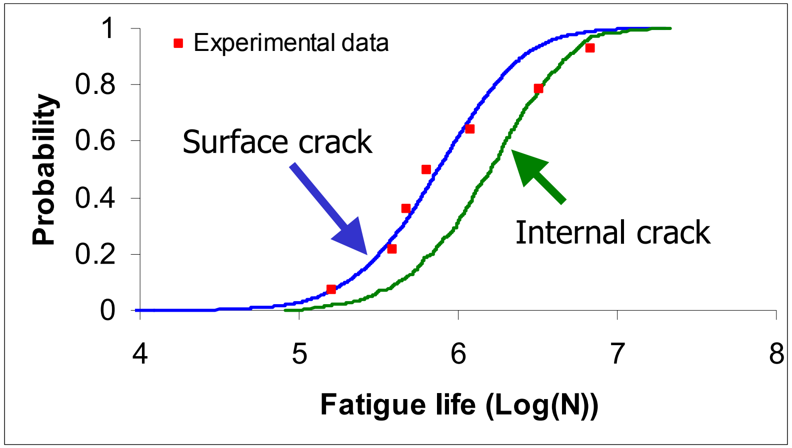
- Plasticity correction considering constraint effect near crack tip

$$a' = a + \rho$$

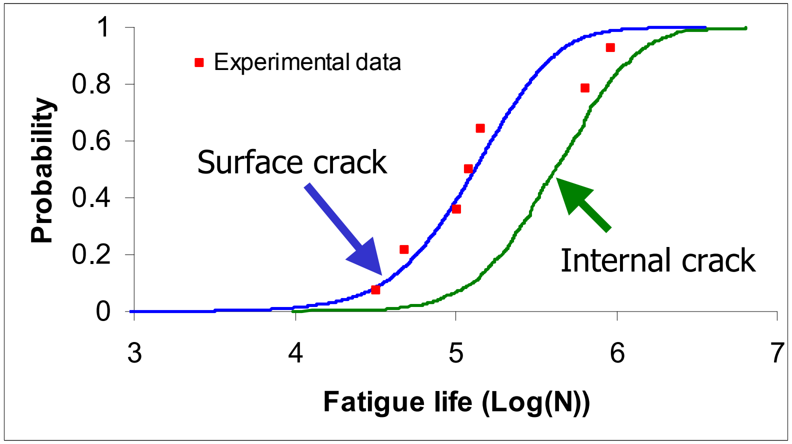
$$\rho = \frac{1}{\alpha} a\left(\sec \frac{\pi \sigma_{max}(1-R)}{4\sigma_0} - 1\right) \qquad \sigma_0 = \left(\frac{\sigma_y + \sigma_u}{2}\right)$$

α : Constraint factor and varies from 1 (plan stress) to 3 (plan strain)
For finite thickness dog-bone specimen, it assumes to be 2 in the current investigation

CDF comparison including internal crack growth



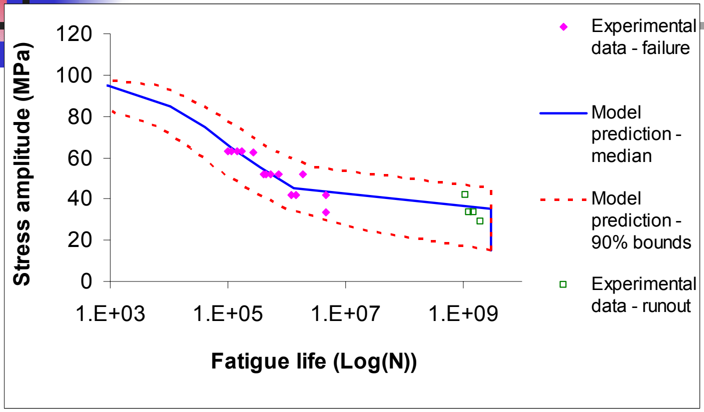
AlSi9Cu3 -20c, S=71MPa



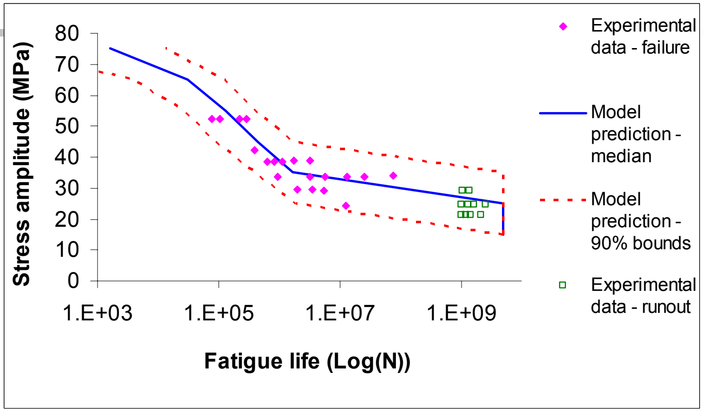
AlSi9Cu3 -150c, S=107MPa

- Internal crack growth mechanism explains the tail shape at long life
- Surface crack growth mechanism explain the tail shape at short life, which is more important for RCDT reliability analysis
- If majority of failure is caused by surface crack propagation, ignoring internal crack growth mechanism will not result in significant error for reliability analysis.

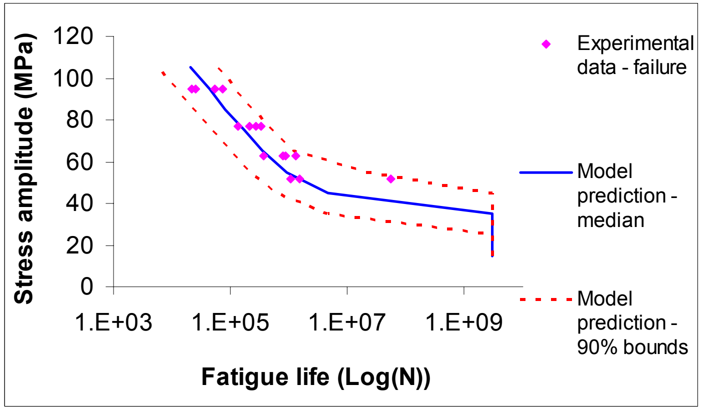
Other materials - 1



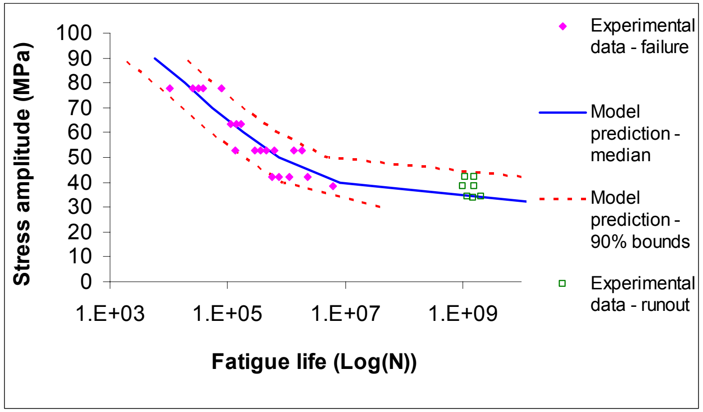
AZ91 hp -20c



AZ91 hp -125c

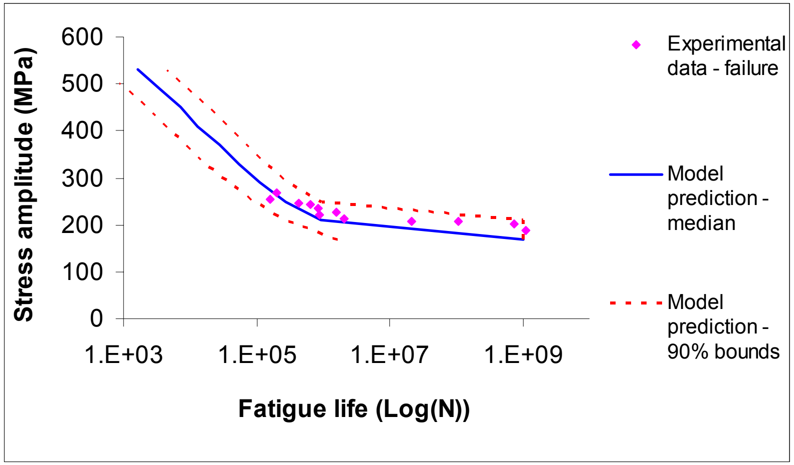


AS21 hp -20c

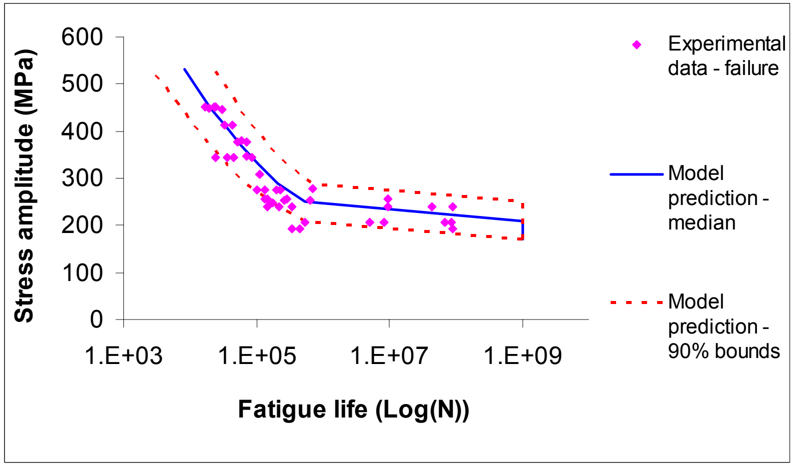


AS21 hp -125c

Other materials - 2



Al 7075-T6 R= -1



Al 7075-T6 R= 0

- Proposed EIFS methodology is effective in predicting fatigue life for the investigated materials and specimens
- Uncertainty quantified and probabilistic crack growth analysis are able to capture the randomness in fatigue life distribution

Current activities and achievements

- Progress from November 1, 2006 – February 1, 2007

    Task 3.1 Uncertainty quantification of FCG properties

    Task 3.2 Probabilistic EIFS calculation

    Task 4.2 Mixed-mode crack propagation

Mixed-mode crack propagation

- Comparison of existing models for mixed-mode crack propagation
- Applicability to RCDT analysis

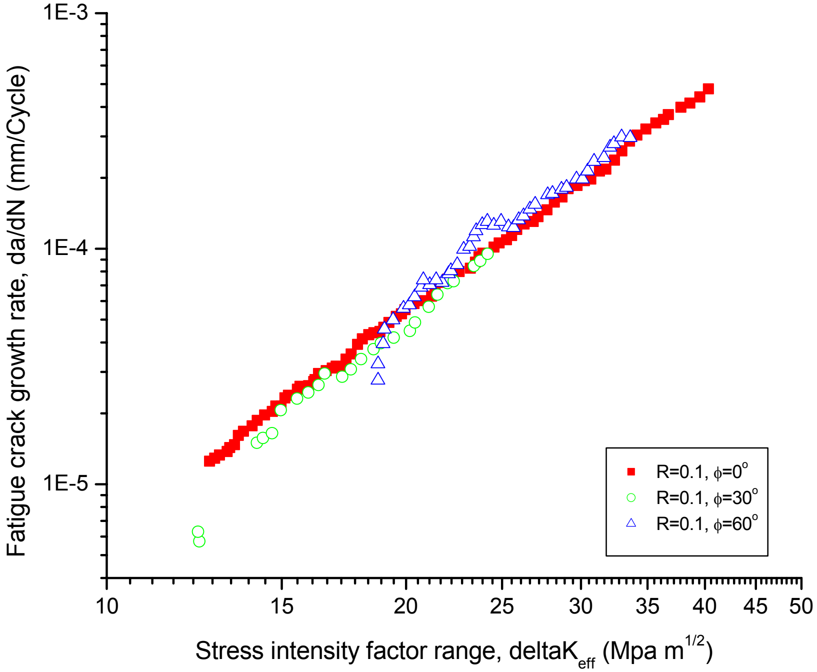
  - Near threshold crack growth

  - High stress ratios
- Characteristic plane-based approach

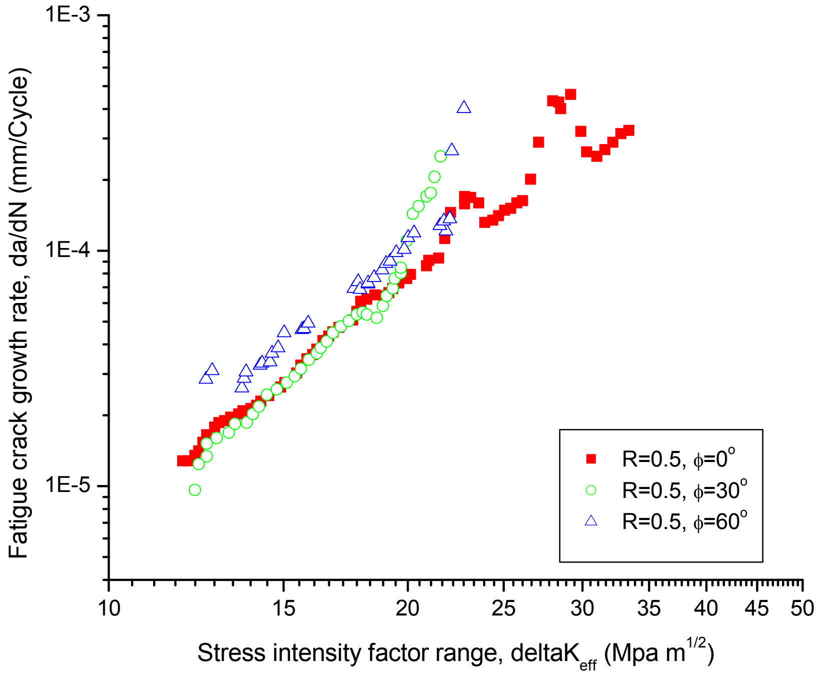
Existing models

- Energy release rate model $$\Delta K_{eff} = \left(\Delta K_I^2 + \Delta K_{II}^2 + \frac{1}{1-\nu} \Delta K_{III}^2 \right)^{\frac{1}{2}}$$
- Strain energy density model $$S = a_{11}k_1^2 + 2a_{12}k_1k_2 + a_{22}k_2^2 + a_{33}k_3^2$$
- Tanaka's model $$\Delta K_{eff} = \left(\Delta K_I^4 + 8\Delta K_{II}^4 \right)^{1/4}$$
- Richard's model $$\left(\frac{K_I}{K_{IC}} \right) + \left(\xi \frac{K_{II}}{K_{IC}} \right)^2 = 1$$
- Tong and Yan's model $$\Delta K_{eff} = \frac{1}{2} \cos \frac{\theta_0}{2} \left[\Delta K_I (1 + \cos\theta_0) - 3\Delta K_{II} \sin\theta_0 \right]$$

Energy release model

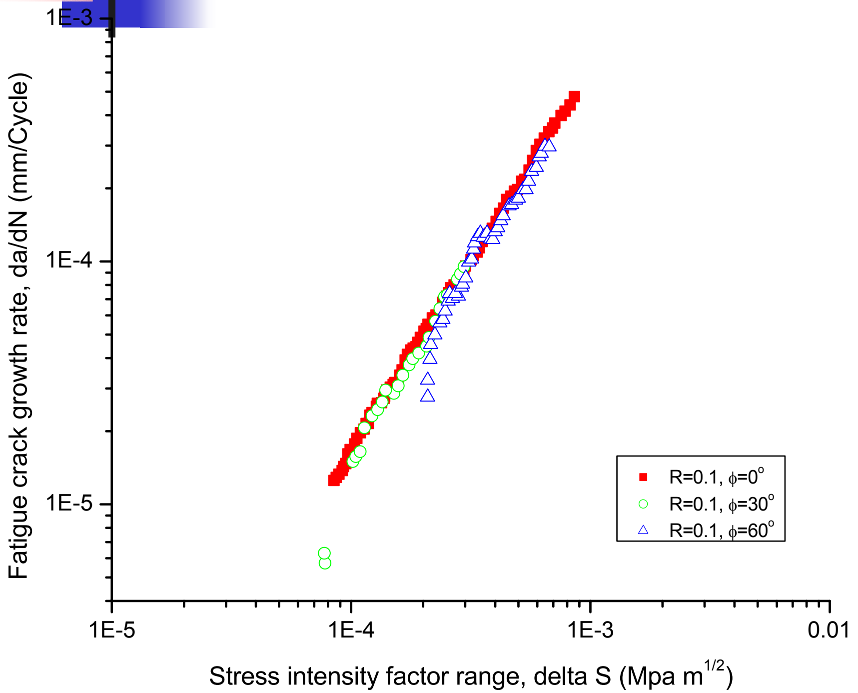


R=0.1

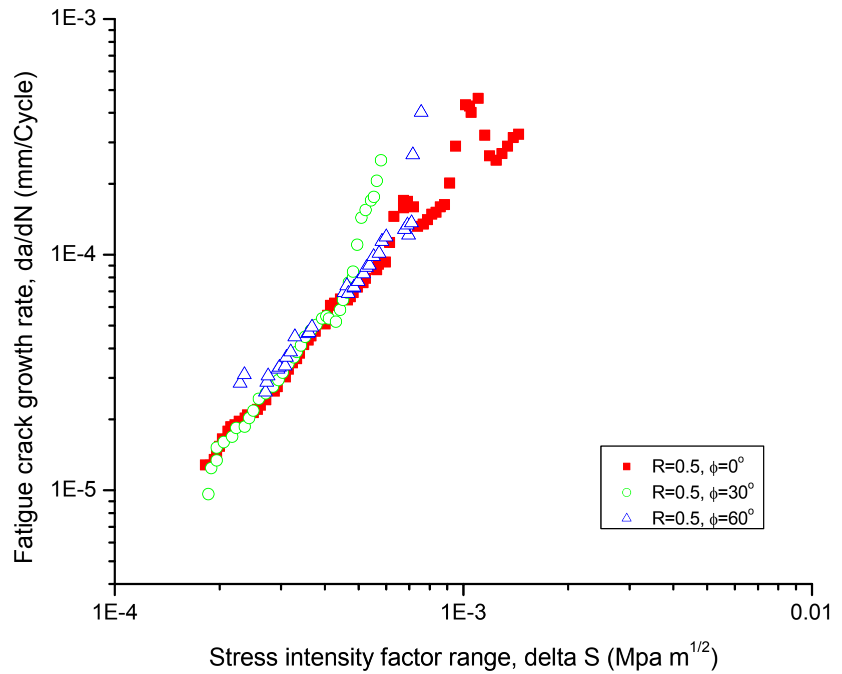


R=0.5

Strain energy density model

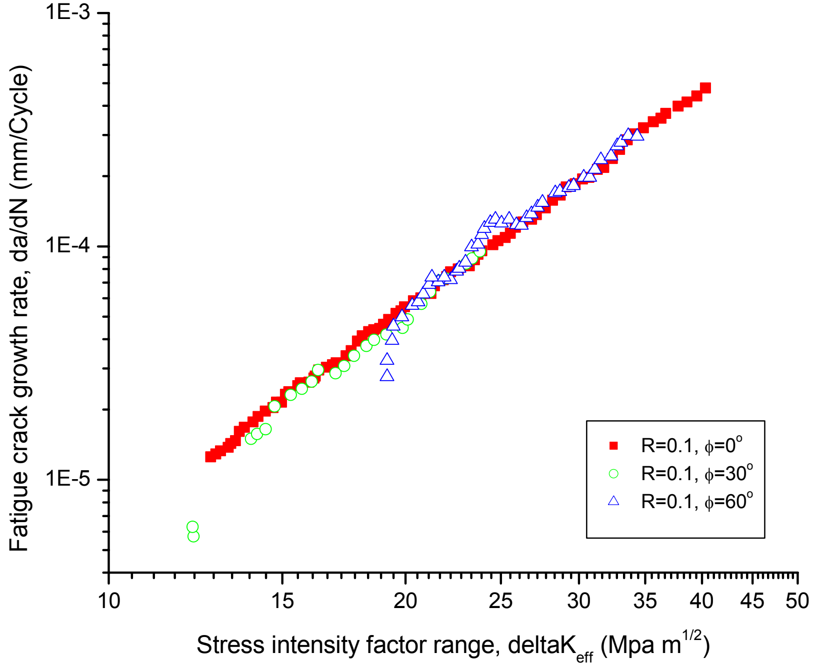


R=0.1

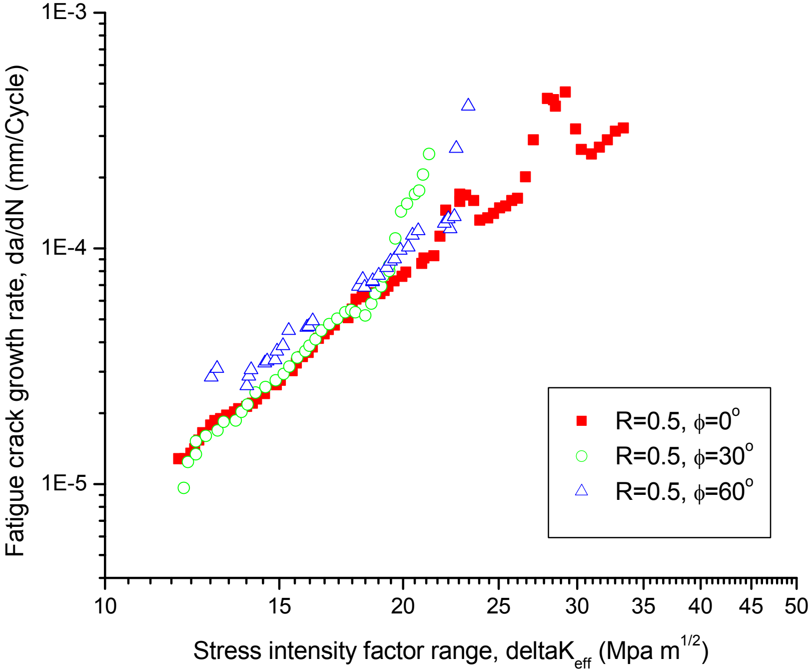


R=0.5

Tanaka's model

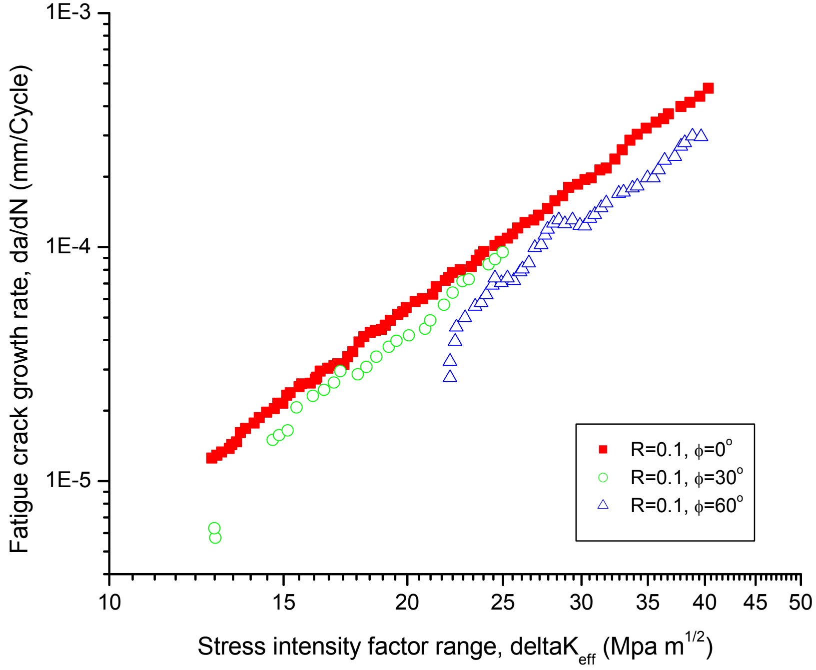


R=0.1

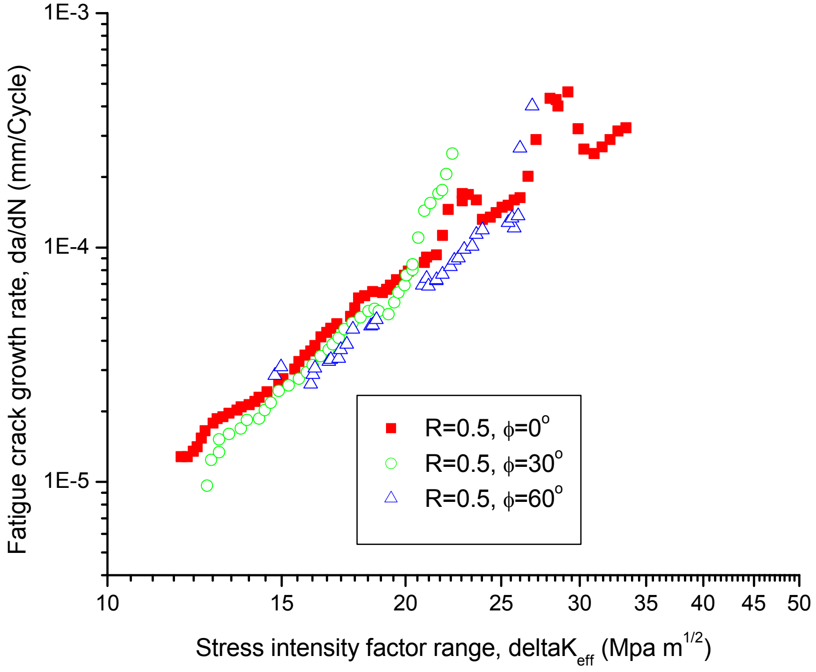


R=0.5

Richard's model

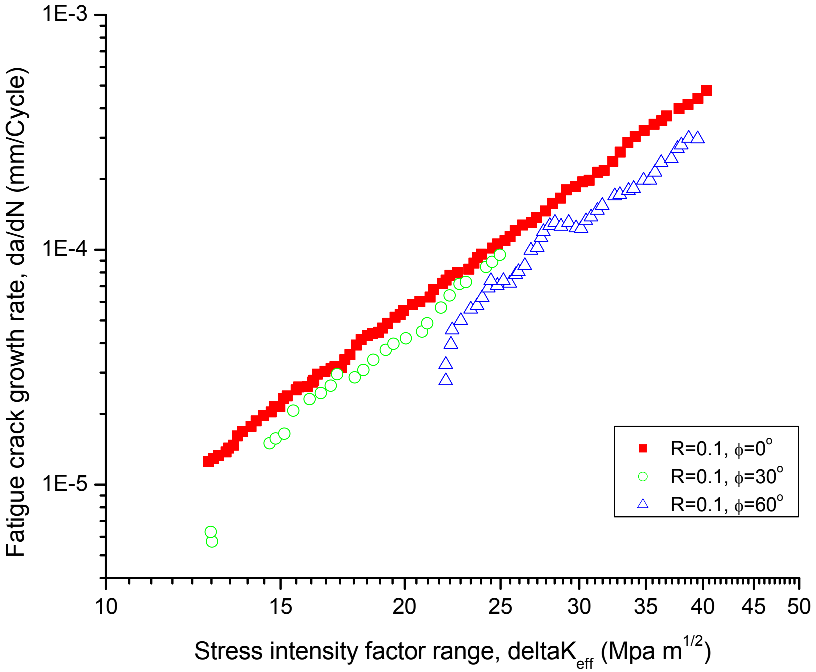


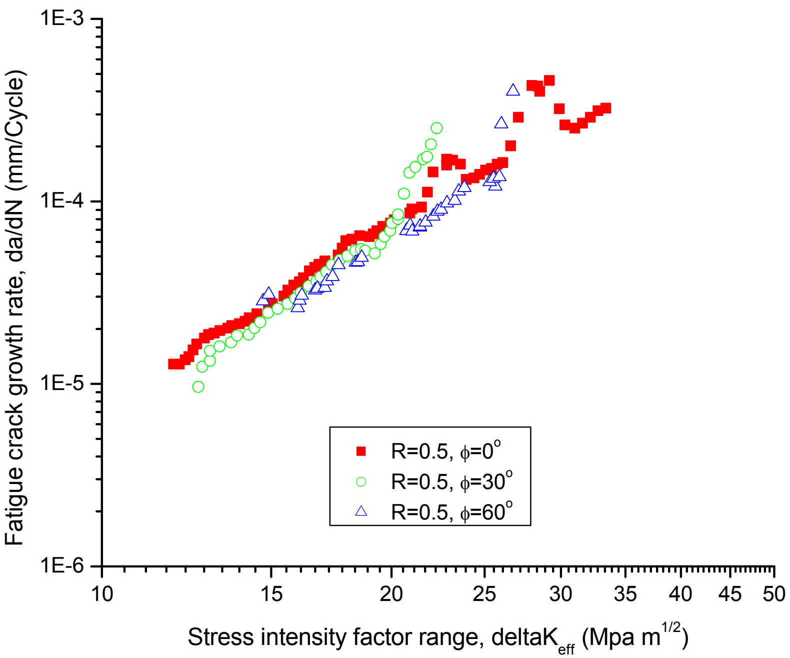
R=0.1



R=0.5

Tong & Yan's model





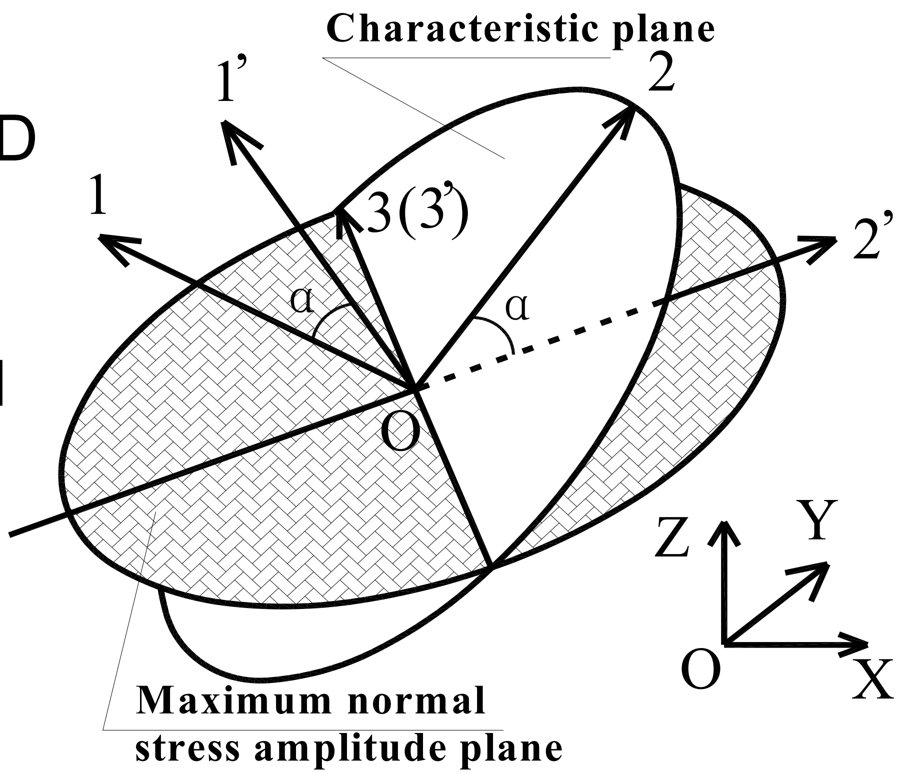
Applicability to RCDT analysis

- Experimental data tends to deviate from prediction at near threshold crack growth
- Failure mechanisms of examined models are Mode I and appropriate for Paris regime growth
- Examined models give better prediction at low stress ratios
- Characteristic plane-based approach for near threshold crack growth

Characteristic plane approach (*C-plane*)

- Mathematical dimension reduction to approximate the 3D problem by 2D components
- The *C-plane* depends on both material properties and applied loads
- New fatigue criterion at fatigue limit stage

$$\sqrt{\left(\frac{\sigma_{a,c}}{f_{-1}}\right)^2 + \left(\frac{\tau_{a,c}}{t_{-1}}\right)^2 + k\left(\frac{\sigma_{a,c}^{H}}{f_{-1}}\right)^2} = \beta$$



Questions?

- How to define the orientation of the characteristic plane?
- How to calculate k and β ?

Two special cases

- Mode I and tensile failure

- Mode II and shear failure

Characteristic plane approach – Case 1

- The characteristic plane is the maximum normal stress amplitude plane (Mode I)

Uniaxial test: $(\sigma_a = f_{-1}, \tau_a = 0)$ **Pure shear test:** $(\sigma_a = 0, \tau_a = t_{-1})$

$$\begin{cases} \sigma_{a,c} = f_{-1} \\ \tau_{a,c} = 0 \\ \sigma_{a,c}^H = f_{-1}/3 \end{cases} \rightarrow \boxed{\sqrt{\left(\frac{\sigma_{a,c}}{f_{-1}}\right)^2 + \left(\frac{\tau_{a,c}}{t_{-1}}\right)^2 + k\left(\frac{\sigma_{a,c}^H}{f_{-1}}\right)^2} = \beta} \leftarrow \begin{cases} \sigma_{a,c} = t_{-1} \\ \tau_{a,c} = 0 \\ \sigma_{a,c}^H = 0 \end{cases}$$

$$\begin{cases} k = 9\left[\left(\frac{t_{-1}}{f_{-1}}\right)^2 - 1\right] \\ \beta = \frac{t_{-1}}{f_{-1}} \end{cases} \longrightarrow \begin{cases} t_{-1}/f_{-1} \geq 1 \\ k \text{ equals zero when } t_{-1}/f_{-1} = 1 \end{cases}$$

Characteristic plane approach – Case 2

- The characteristic plane is the maximum shear stress amplitude plane (Mode II)

Uniaxial test: $(\sigma_a = f_{-1}, \tau_a = 0)$

$$\begin{cases} \sigma_{a,c} = f_{-1}/2 \\ \tau_{a,c} = f_{-1}/2 \\ \sigma_{a,c}^{H} = f_{-1}/3 \end{cases}$$

Pure shear test: $(\sigma_a = 0, \tau_a = t_{-1})$

$$\begin{cases} \sigma_{a,c} = 0 \\ \tau_{a,c} = t_{-1} \\ \sigma_{a,c}^{H} = 0 \end{cases}$$

$$\boxed{\sqrt{\left(\frac{\sigma_{a,c}}{f_{-1}}\right)^2 + \left(\frac{\tau_{a,c}}{t_{-1}}\right)^2 + k\left(\frac{\sigma_{a,c}^{H}}{f_{-1}}\right)^2} = \beta}$$

$$\begin{cases} k = \dfrac{3}{2}\sqrt{3 - \left(\dfrac{f_{-1}}{t_{-1}}\right)^2} \\ \beta = 1 \end{cases} \longrightarrow \begin{cases} t_{-1}/f_{-1} \geq 1/\sqrt{3} \\ k \text{ equals zero when } f_{-1}/t_{-1} = \sqrt{3} \end{cases}$$

Conclusions from the two cases

- If the characteristic plane is fixed, the range of applicable materials is limited
- The contribution of the hydrostatic stress amplitude is zero for two types of materials

A general model:

- let the characteristic plane to rotate
- minimize the contribution of hydrostatic stress amplitude to zero

Characteristic plane approach – general model

- The characteristic plane is α degree off the maximum normal stress plane

Uniaxial test: $(\sigma_a = f_{-1}, \tau_a = 0)$

$$\begin{cases} \sigma_{a,\alpha} = \dfrac{f_{-1}}{2} \pm \dfrac{f_{-1}}{2} \cos(2\alpha) \\ \tau_{a,\alpha} = \pm \dfrac{f_{-1}}{2} \sin(2\alpha) \end{cases}$$

Pure shear test: $(\sigma_a = 0, \tau_a = t_{-1})$

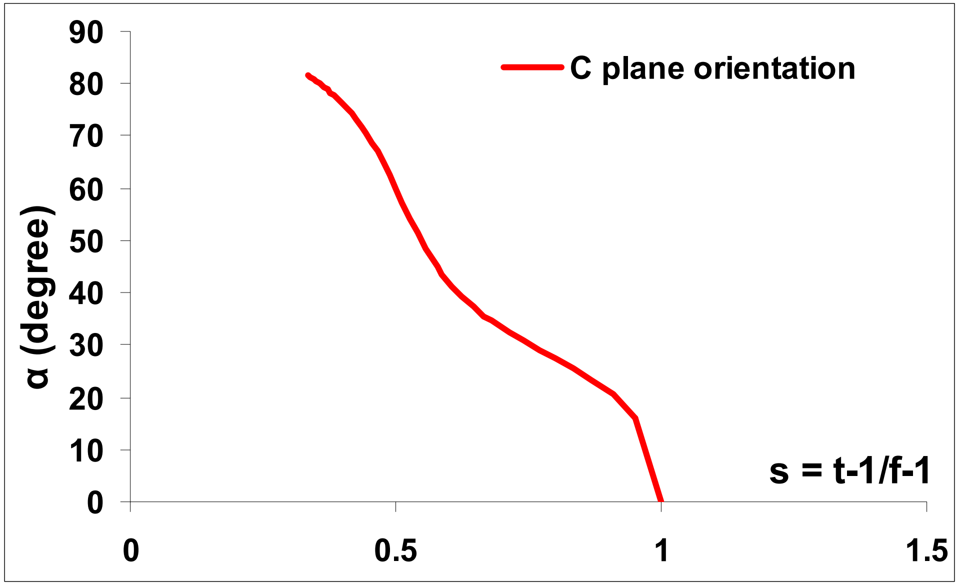
$$\begin{cases} \sigma_{a,\alpha} = \pm t_{-1} \cos(2\alpha) \\ \tau_{a,\alpha} = \pm t_{-1} \sin(2\alpha) \end{cases}$$

$$\boxed{\sqrt{\left(\frac{\sigma_{a,\alpha}}{f_{-1}}\right)^2 + \left(\frac{\tau_{a,\alpha}}{t_{-1}}\right)^2} = \beta}$$

$$\begin{cases} \cos(2\alpha) = \dfrac{-2 + \sqrt{4 - 4(1/s^2 - 3)(5 - 1/s^2 - 4s^2)}}{2(5 - 1/s^2 - 4s^2)} \\ \beta = [\cos^2(2\alpha)s^2 + \sin^2(2\alpha)]^{\frac{1}{2}} \end{cases}$$

$$s = \frac{t_{-1}}{f_{-1}}$$

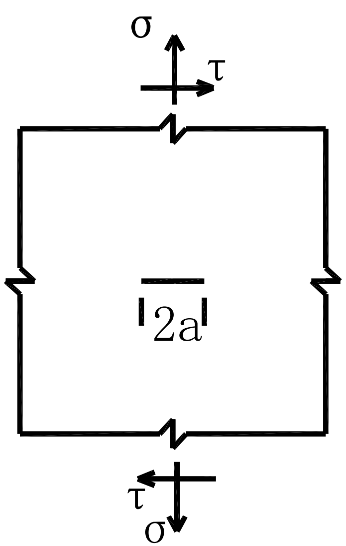
Characteristic plane approach - 6



s value relates to failure modes

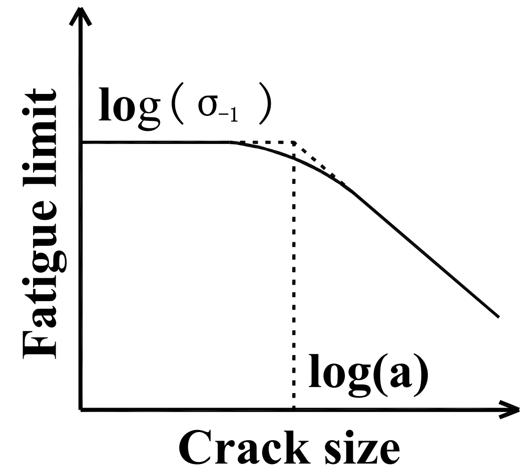
Larger *s* – (Mode I) tensile failure

Fatigue limit and threshold stress intensity factor



Mixed-mode threshold stress intensity factor

$$\sqrt{(\frac{k_1}{K_{I,th}})^2+(\frac{k_2}{K_{II,th}})^2+A(\frac{k^H}{K_{I,th}})^2}=B$$



Kitagawa diagram

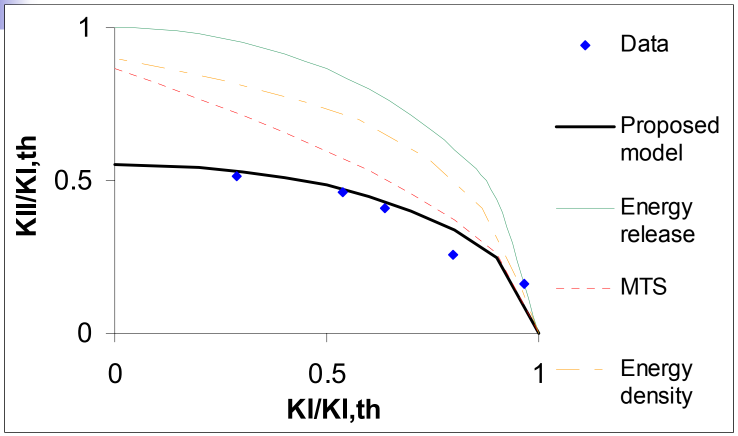
Multiaxial fatigue limit criterion

$$\sqrt{(\frac{\sigma_c}{f_{-1}})^2+(\frac{\tau_c}{t_{-1}})^2+A(\frac{\sigma^H}{f_{-1}})^2}=B$$

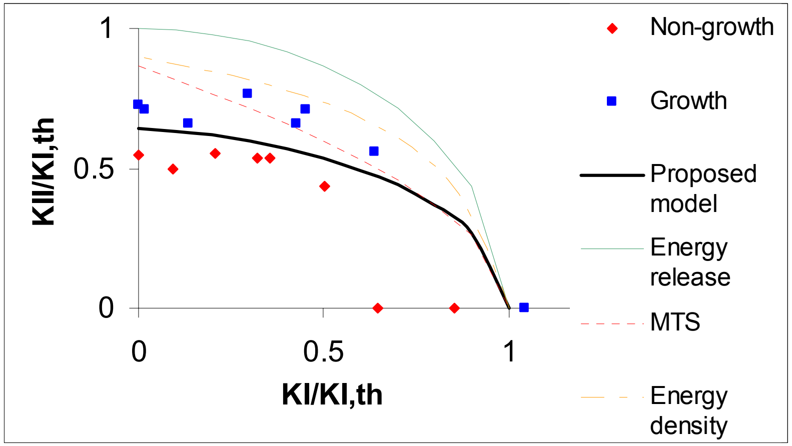
El Haddad model

$$f_{-1}=\frac{K_{I,th}}{\sqrt{\pi a}} \qquad t_{-1}=\frac{K_{II,th}}{\sqrt{\pi a}}$$

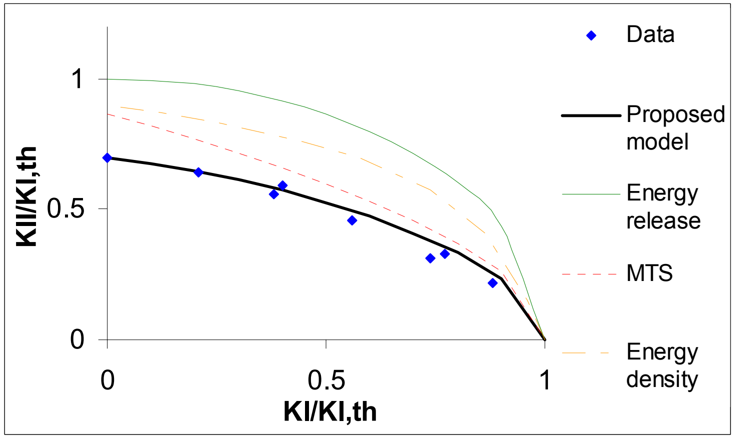
Threshold prediction 1



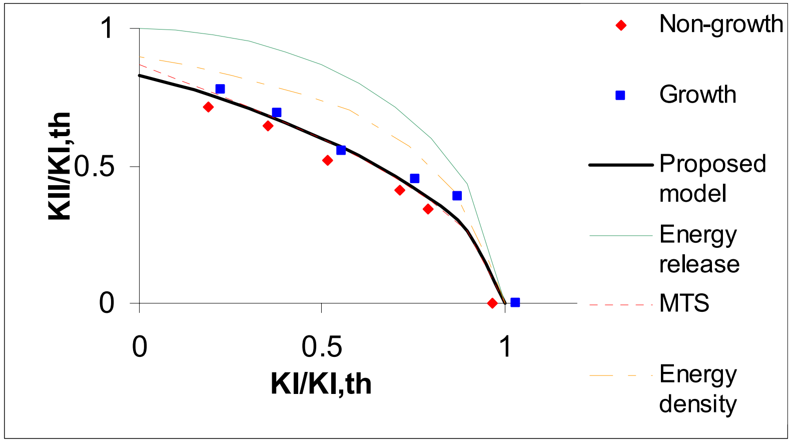
6061Al



7075-T6 aluminum

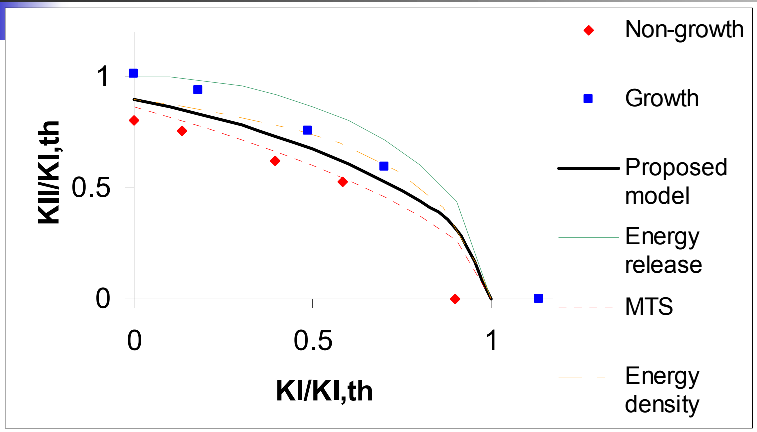


316 stainless steel

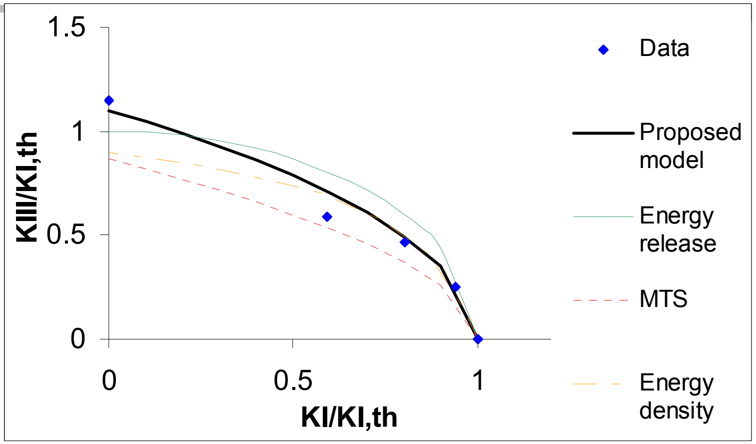


Aluminum alloy

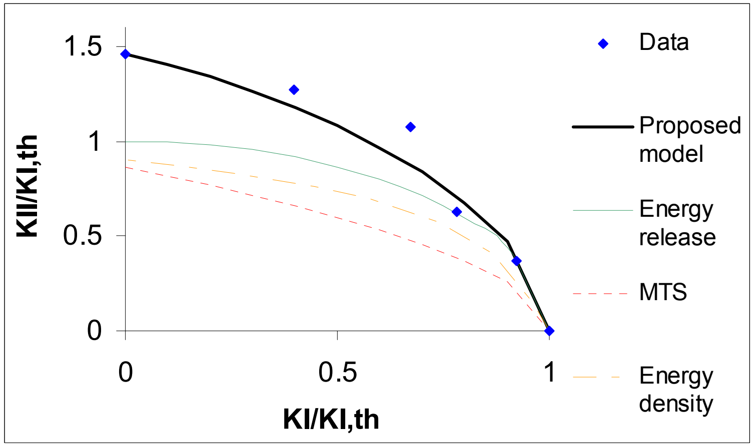
Threshold prediction 2



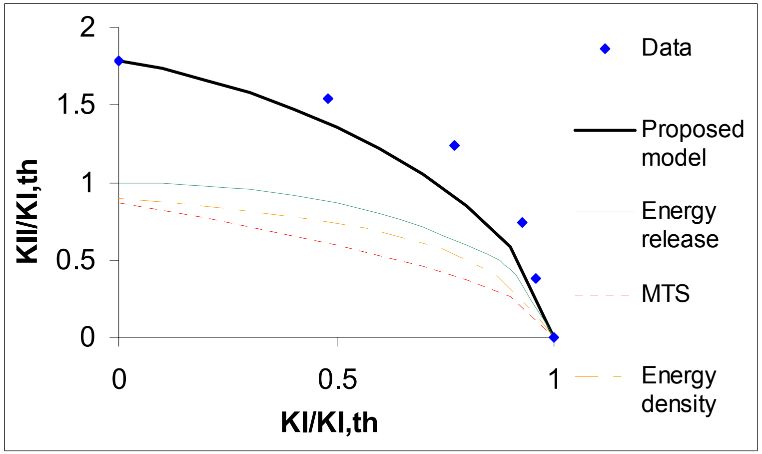
2017-T3 aluminum



Mild steel

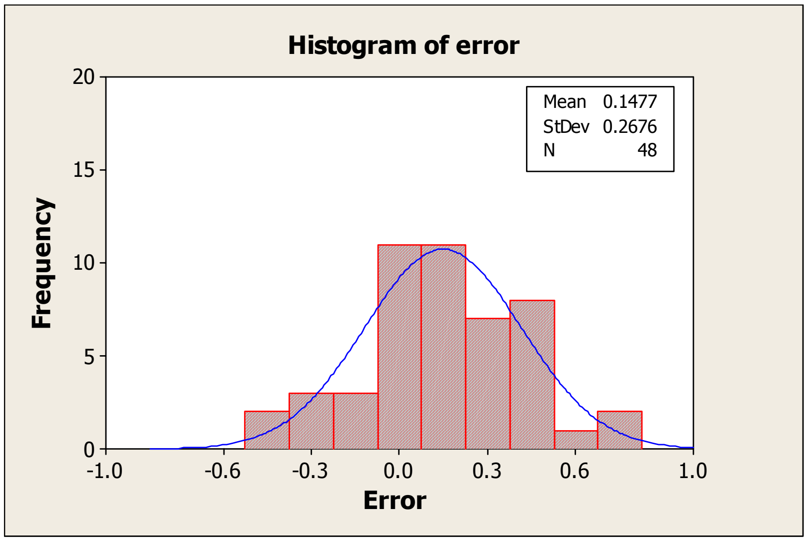


2024 Al

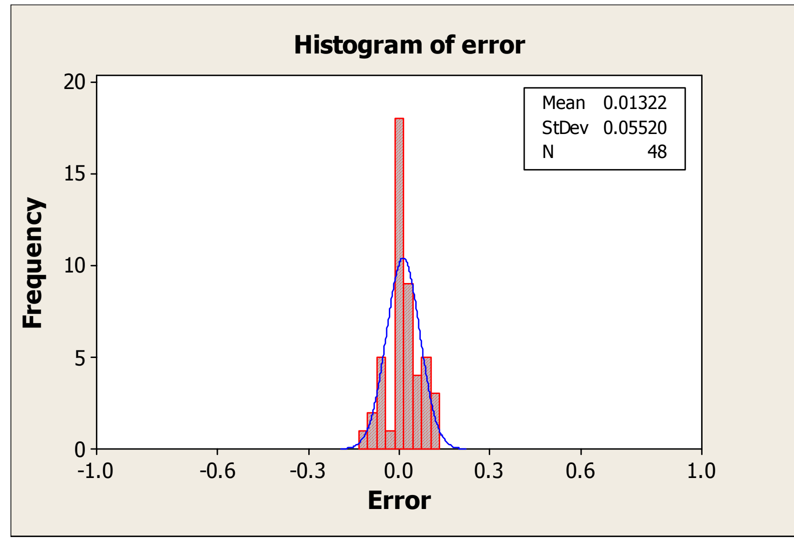


SiCp/2024Al composite

Modeling model uncertainty



Energy release rate model



Characteristic plane-based approach

Error=(prediction-observation)/observation

Conclusions

- Data collection and analysis for uncertainty quantification of FCG properties
- General methodology for probabilistic EIFS calculation
- Preliminary probabilistic crack growth analysis including plasticity correction and internal crack growth mechanism
- Demonstration example and validation of the proposed EIFS methodology
- Model comparison of different mixed-mode fatigue crack models; improvements are required for RCDT analysis
- Characteristic plane-based approach for near threshold crack growth

Planned work (next 6 months) - 1

- Task 3 Uncertainty quantification

  - Continue to collect and analyze FCG properties

  - Develop and compare different models in representing data

  - Quantify associated modeling uncertainty

  - Start to collect and analyze load spectrum data

  - Start to quantify uncertainties in applied loads using both frequency domain and time domain approaches

Planned work - 2

- Task 3 Uncertainty quantification

  - Improve the accuracy of the proposed probabilistic EIFS methodology, especially for the correlation effect of random variables

  - Continue and quantify EIFS distribution for other types of material for rotorcrafts

  - Demonstrate and validate the proposed EIFS methodology for different materials and specimen configuration

Planned work - 3

- Task 3 Uncertainty propagation

  - Develop a general probabilistic crack growth analysis methodology

  - Investigate the effect of plasticity correction factor and notch effect

  - Continue to investigate mixed-mode load effect on near threshold crack growth behavior

  - Compare and quantify modeling errors in uncertainty propagation through FCG analysis

Planned work - 4

- Task 3 Uncertainty propagation

  - Start to investigate analytical approximations and multi-resolution computation approaches in calculating stress intensity factors

  - Start to quantify associated uncertainties with numerical computation method, e.g. mesh density, element type and shape, boundary conditions.

Planned work - 5

- Task 5 Risk assessment

  - Start to investigate efficient methods in calculating time-dependent fatigue reliability, e.g. FORM (first order reliability method) and MC (Monte Carlo) method

  - Demonstrate simple examples using results obtained in Task 3 and 4